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$$\left| f - \sum_{q=1}^{2n+1} h(t_q) \right| \leq (n+1)\rho + n \frac{2}{2n+1} \|f\|.$$

But  $\lim_{\delta \rightarrow 0} \rho = 0$ , consequently for sufficiently small  $\delta$  and  $\varepsilon < \frac{1}{2n+2}$

$$\left| f - \sum_{q=1}^{2n+1} h(t_q) \right| < (1-\varepsilon) \|f\|.$$

The lemma is proved.

#### § 4. *The proof of the theorem*

We denote by  $F$  a countable set, everywhere dense in  $C(\mathcal{I}^n)$ . We choose  $\varepsilon$  satisfying the condition of lemma 3.3.1 and consider  $\Omega_{f_k}$  ( $f_k \in F$ ) corresponding to this  $\varepsilon$  and the collection  $\lambda_p$  mentioned in the theorem. The sets  $\{\Omega_{f_k}\}$  are open and by lemma 3.3.1 they are everywhere dense in  $\Phi^{2n+1}$ . Consequently, according to the definition, almost every element of  $\Phi^{2n+1}$  belongs to  $\Phi^* = \bigcap_{f_k \in F} \Omega_{f_k}$ .

We fix a collection  $\{\varphi_1, \dots, \varphi_{2n+1}\} \in \Phi^*$  and a function  $f \in C(\mathcal{I}^n)$  and show that the desired representation of  $f$  takes place. If  $f \equiv 0$  then as the function  $g$  we can take  $g \equiv 0$ . We will assume below that  $f \not\equiv 0$ . According to the definition of  $\Omega_{f_k}$  there exists for any  $f_k \in F$  a function  $h_k$  such that

$\left| f_k - \sum_{q=1}^{2n+1} h_k \left( \sum_{p=1}^n \lambda_p \varphi_q(x_p) \right) \right| \leq (1-\varepsilon) \|f_k\|$ . The set  $F$  is everywhere dense in  $C(\mathcal{I}^n)$ . Consequently for any  $f \in C(\mathcal{I}^n)$  ( $f \not\equiv 0$ ) there exists  $h = \gamma(f)$  such that

$$\left| f - \sum_{q=1}^{2n+1} h \left( \sum_{p=1}^n \lambda_p \varphi_q(x_p) \right) \right| < \left( 1 - \frac{\varepsilon}{2} \right) \|f\|.$$

We define the sequence of functions  $\chi_0, \chi_1, \chi_2, \dots$  by the recurrent equalities

$$\chi_0 = f, \quad \chi_{k+1} = \chi_k - \sum_{q=1}^{2n+1} g_k \left( \sum_{p=1}^n \lambda_p \varphi_q(x_p) \right),$$

where  $g_k = \gamma(\chi_k)$ . The series  $\sum_{k=0}^{\infty} g_k$  converges uniformly and consequently the function  $g = \sum_{k=0}^{\infty} g_k$  is continuous and

$$f - \sum_{q=1}^{2n+1} g \left( \sum_{p=1}^n \lambda_p \varphi_q(x_p) \right) = 0.$$

The theorem is proved.