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$$|f - \sum_{q=1}^{2n+1} h(t_q)| \leqslant (n+1)\rho + n\frac{2}{2n+1}\|f\| .$$

But $\lim\limits_{\delta\to 0} \rho = 0$, consequently for sufficiently small δ and $\varepsilon < \dfrac{1}{2n+2}$

$$|f - \sum_{q=1}^{2n+1} h(t_q)| < (1-\varepsilon)\|f\| .$$

The lemma is proved.

§ 4. *The proof of the theorem*

We denote by F a countable set, everywhere dense in $C(\mathscr{I}^n)$. We choose ε satisfying the condition of lemma 3.3.1 and consider Ω_{f_k} $(f_k \in F)$ corresponding to this ε and the collection λ_p mentioned in the theorem. The sets $\{\Omega_{fk}\}$ are open and by lemma 3.3.1 they are everywhere dense in Φ^{2n+1}. Consequently, according to the definition, almost every element of Φ^{2n+1} belongs to $\Phi^* = \bigcap\limits_{f_k \in F} \Omega_{f_k}$.

We fix a collection $\{\varphi_1, \ldots, \varphi_{2n+1}\} \in \Phi^*$ and a function $f \in C(\mathscr{I}^n)$ and show that the desired representation of f takes place. If $f \equiv 0$ then as the function g we can take $g \equiv 0$. We will assume below that $f \not\equiv 0$. According to the definition of Ω_{f_k} there exists for any $f_k \in F$ a function h_k such that $|f_k - \sum\limits_{q=1}^{2n+1} h_k(\sum\limits_{p=1}^{n} \lambda_p \varphi_q(x_p))| \leqslant (1-\varepsilon)\|f_k\|$. The set F is everywhere dense in $C(\mathscr{I}^n)$. Consequently for any $f \in C(\mathscr{I}^n)$ $(f \not\equiv 0)$ there exists $h = \gamma(f)$ such that

$$|f - \sum_{q=1}^{2n+1} h(\sum_{p=1}^{n} \lambda_p \varphi_q(x_p))| < \left(1 - \frac{\varepsilon}{2}\right)\|f\| .$$

We define the sequence of functions $\chi_0, \chi_1, \chi_2, \ldots$ by the recurrent equalities

$$\chi_0 = f, \quad \chi_{k+1} = \chi_k - \sum_{q=1}^{2n+1} g_k(\sum_{p=1}^{n} \lambda_p \varphi_q(x_p)),$$

where $g_k = \gamma(\chi_k)$. The series $\sum\limits_{k=0}^{\infty} g_k$ converges uniformly and consequently the function $g = \sum\limits_{k=0}^{\infty} g_k$ is continuous and

$$f - \sum_{q=1}^{2n+1} g(\sum_{p=1}^{n} \lambda_p \varphi_q(x_p)) = 0 .$$

The theorem is proved.