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**Autor:** Vitushkin, A. G.  
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# ON REPRESENTATION OF FUNCTIONS BY MEANS OF SUPERPOSITIONS AND RELATED TOPICS <sup>1</sup>

by A. G. VITUSHKIN

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<sup>1</sup>) Summary of lectures given at the University of California in Los Angeles, in April-May 1977, under the sponsorship of the International Mathematical Union.

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### PREFACE

By means of an algebraic substitution, the so-called Tschirnhaus transformation, the general algebraic equation of the  $n$ -th degree  $x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n = 0$  may be reduced to the form  $y^n + b_4 y^{n-4} + b_5 y^{n-5} + \dots + b_{n-1} y + 1 = 0$ . Further attempts by algebraists to reduce the solution of the general algebraic equation to the solution of equations containing a smaller number of parameters remained unsuccessful for a long time (the problem of resolvents).

In his famous Mathematical Problems [1] Hilbert looked at this problem in a new way, formulating it as No. 13 in the following form: the impossibility of solving the general equation of the 7-th degree by means of functions of only two variables. To prove this Hilbert regarded it as possible to show that the equation of the 7-th degree  $f^7 + xf^3 + yf^2 + zf + 1 = 0$  is not soluble by means of any continuous functions of only two variables.

Various mathematicians have understood the 13-th Problem differently and have attributed to it results of a different character.

Hilbert [3] found an algebraic substitution reducing the solution of the general algebraic equation of the 9-th degree to the solution of equations with 4 parameters. Hilbert proved also the existence of analytic functions of three variables not representable by superpositions of functions of only two variables. Ostrowski [2] constructed an analytic function of two variables not representable as a superposition of infinitely differentiable functions of one variable and arithmetic operations. The author [4] proved the