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§5. EXAMPLES

Given an elliptic curve E in the form of a minimal model (1.1) or (1.2), one computes the bad primes by finding the prime divisors of the discriminant Δ . We can then apply the methods of the preceding sections to determine f_p and hence the type of reduction.

Example 5.1. Let E be given by $Y^2 = X^3 + X + 1$. This equation is minimal. The discriminant is $\Delta = -16(31)$, so E has bad reduction at $p = 2$ and $p = 31$. For $p = 2$, $C_{p-1} = C_1 = a_1 = 0$ so we have additive reduction at $p = 2$. For $p = 31$, we can apply Theorem 4.3 and Corollary 4.4. $f_p = \left(\frac{-2AB}{p}\right) = \left(\frac{-2}{31}\right) = -1$, so that E has non-split multiplicative reduction at $p = 31$. Alternatively, one may use Deuring's formula to compute C_{p-1} . A third possibility, of course, is to factor $X^3 + X + 1$ over $\mathbf{Z}/31\mathbf{Z}$ and then analyse (4.14). $c_4 = -48$.

Example 5.2. Let E be given by $Y^2 = X^3 + X - 1$. The equation is minimal and $\Delta = -16(31)$. We have additive reduction at $p = 2$ since $C_{p-1} = C_1 = a_1 = 0$. For $p = 31$, $f_p = \left(\frac{-2AB}{p}\right) = \left(\frac{2}{31}\right) = 1$, so that E has split multiplicative reduction at $p = 31$. $c_4 = -48$.

Remark. Comparing examples 5.1 and 5.2, one sees that c_4 is the same in both cases. However, 5.1. exhibits non-split multiplicative reduction at $p = 31$, while 5.2 exhibits split multiplicative reduction at the same prime.

Example 5.3. Let E be given by $Y^2 = X^3 + 7X + 5$. The equation is minimal and $\Delta = -16(23)(89)$. E has bad reduction at $p = 2, 23$, and 89 . For $p = 2$, $C_{p-1} = C_1 = a_1 = 0$, so we have additive reduction at $p = 2$. For $p = 23$, we have $f_p = \left(\frac{-2AB}{p}\right) = \left(\frac{-70}{23}\right) = \left(\frac{-1}{23}\right) = -1$, so that E has non-split multiplicative reduction at $p = 23$. For $p = 89$, we have $f_p = \left(\frac{-2AB}{p}\right) = \left(\frac{19}{89}\right) = -1$, so that E has non-split multiplicative reduction at $p = 89$ as well.

Remark. The computation of the Legendre symbol is much easier to carry out in practice than either the computation of C_{p-1} via Deuring's formula or by searching for roots of the polynomial $X^3 + AX + B$.

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