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ANALOGIES WITH FOURIER SERIES

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THEOREM 4.5. *If $K^{(s)}(x, y)$ belongs both to $\text{Lip}(\alpha, p)$ and to $\text{Lip} \beta$, then $\sum (1/\mu_n)^\gamma$ converges for all $\gamma > \rho$ where ρ is as given in Theorem 2.10.*

Naturally, these theorems also contain the analogues of the Zygmund and Waraszkiewicz results, Theorems 2.4, 2.5.

In closing it is worth remarking that all of the above kernel function results are equally as sharp as the corresponding Fourier series results since, as we have seen earlier, for periodic difference kernels the singular values and the related Fourier coefficients are essentially reciprocals. In view of the Weyl-Chang inequalities (4.2), moreover, these theorems amplify and extend our knowledge concerning the growth behavior of the *characteristic* values of “smooth” kernels (see [22], [11], for example).

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