Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 22 (1976)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SUMMABILITY OF SINGULAR VALUES OF L^2 KERNELS. ANALOGIES WITH FOURIER SERIES

Autor: Cochran, James Alan

Bibliographie

DOI: https://doi.org/10.5169/seals-48180

THEOREM 4.5. *If* $K^{(s)}(x, y)$ *belongs both to* Lip (α, p) *and to* Lip β, *then* $\sum (1/\mu_n)^\gamma$ *converges for all* $\gamma > \rho$ *where* ρ *is as given in Theorem 2.10.*

Naturally, these theorems also contain the analogues of the Zygmund and Waraszkiewicz results, Theorems 2.4, 2.5.

In closing it is worth remarking that all of the above kernel function results are equally as sharp as the corresponding Fourier series results since, as we have seen earlier, for periodic difference kernels the singular values and the related Fourier coefficients are essentially reciprocals. In view of the Weyl-Chang inequalities (4.2), moreover, these theorems amplify and extend our knowledge concerning the growth behavior of the *characteristic* values of "smooth" kernels (see [22], [11], for example).

5. ACKNOWLEDGEMENT

Our original interest in this entire inquiry owed much to numerous stimulating discussions with our former colleague D. W. Swann.

REFERENCES

[1] BARY, N. K. *A treatise on trigonometric series.* Vol. II (translated from the 1961 Russian ed. by M. F. Mullins). Pergamon, New York, 1964.

[2] BERNSTEIN, S. N. Sur la convergence absolue des séries trigonométriques. *C.R. Acad. Sci. Paris 158* (1914), pp. 1661-1663.

[3] —— On the absolute convergence of trigonometric series. *Soobshch. Khar'kov. Mat. Obshch. (2) 14* (1914), pp. 139-144 (in Russian); *ibid.* (1915) pp. 200-201; see also *Collected Works*, Vol. I, 1952, pp. 217-223 (in Russian).

[4] —— Sur la convergence absolue des séries trigonométriques. *C.R. Acad. Sci. Paris 199* (1934), pp. 397-400.

[5] CARLEMAN, T. Über die Fourierkoeffizienten einer stetigen Funktion. *Acta Math. 41* (1918), pp. 377-384; see also *Edition complète des articles*, Malmö, 1960, pp. 15-22.

[6] —— Zur Theorie der linearen Integralgleichungen. *Math. Zeit. 9* (1921), pp. 196-217; see also *Edition complète des articles,* Malmö 1960, pp. 79-100.

[7] CHANG, S. H. A generalization of a theorem of Lalesco. *J. London Math. Soc. 22* (1947), pp. 185-189.

[8] —— On the distribution of the characteristic values and singular values of linear integral equations. *Trans. Amer. Math. Soc. 67* (1949), pp. 351-367. MR 11, 523.

[9] —— A generalization of a theorem of Hille and Tamarkin with applications. *Proc. London Math. Soc. (3) 2* (1952), pp. 22-29. MR 13, 950.

[10] COCHRAN, J. A. The existence of eigenvalues for the integral equations of laser theory. *Bell Syst. Tech. J. 44* (1965), pp. 77-88. MR 30. 1368.

[11] —— *The analysis of linear integral equations.* McGraw-Hill, New York, 1972.

[12] —— The nuclearity of operators generated by Hölder continuous kernels. *Proc. Cambridge Phil. Soc. 75* (1974), pp. 351-356.

[13] —— Growth estimates for the singular values of square-integrable kernels. *Pacific J. Math. 56* (1975), pp. 51-58.

[14] FAN, K. Maximum properties and inequalities for the eigenvalues of completely continuous operators. *Proc. Nat. Acad. Sci. U.S.A. 37* (1951), pp. 760-766.

[15] FREDHOLM, I. Sur une classe d'équations fonctionnelles. *Acta Math. 27* (1903), pp. 365-390; see also *Oeuvres complètes*, Malmö, 1955, pp. 81-106.

[16] GOHBERG, I. C. and M. G. KREIN. Introduction to the theory of linear nonselfadjoint operators. *Translations of Math. Mono., vol. 18, Amer. Math. Soc.*, Providence, R.I., 1969.

[17] HARDY, G. H. and J. E. LITTLEWOOD. Some new properties of Fourier constants. *Math. Ann. 97* (1926), pp. 159-209; see also *Collected papers of G. H. Hardy*, vol. III, Clarendon, Oxford, 1969, pp. 348-399.

[18] —— Some properties of fractional integrals. I. *Math. Zeit. 27* (1928), pp. 565-606; see also *Collected papers of G. H. Hardy, op. cit.*, 1969, pp. 564-607.

[19] —— A convergence criterion for Fourier series. *Math. Zeit 28* (1928), pp. 612-634; see also *Collected papers of G. H. Hardy, op. cit.*, 1969, pp. 28-51.

[20] —— Notes on the theory of series (IX): On the absolute convergence of Fourier series. *J. London Math. Soc. 3* (1928), pp. 250-253; see also *Collected papers of G. H. Hardy, op. cit.*, 1969, pp. 52-56.

[21] HAUSDORFF, F. Eine Ausdehnung des Parsevalschen Satzes über Fourierreihen. *Math. Zeit. 16* (1923), pp. 163-169.

[22] HILLE, E. and J. D. TAMARKIN. On the characteristic values of linear integral equations. *Acta Math. 57* (1931), pp. 1-76.

[23] SCHUR, I. Über die charakteristischen Wurzeln einer linearen Substitution mit einer Anwendung auf die Theorie der Integralgleichungen. *Math. Ann. 66* (1909), pp. 488-510.

[24] SMITHIES, F. The eigen-values and singular values of integral equations. *Proc. London Math. Soc. (2) 43* (1937), pp. 255-279.

[25] —— *Integral equations*. Cambridge, London, 1962, pp. 148-150.

[26] SZÁSZ, O. Über den Konvergenzexponenten der Fourierschen Reihen gewisser Funktionenklassen. *Sitzungsber. Akad. Wiss. München Math. Phys. Kl.* (1922), pp. 135-150; see also *Collected mathematical papers*, Dept. of Math., Cincinnati, Cincinnati, Ohio, 1955, pp. 684-699.

[27] —— Über die Fourierschen Reihen gewisser Funktionenklassen. *Math. Ann. 100* (1928), pp. 530-536; see also *Collected mathematical papers, op. cit.*, 1955, pp. 758-764.

[28] TITCHMARSH, E. C. A note on Fourier transforms. *J. London Math. Soc. 2* (1927), pp. 148-150.

[29] —— *Introduction to the theory of Fourier integrals.* 2nd. ed., Clarendon, Oxford, 1948, pp. 115-117.

[30] TONELLI, L. Sulla convergenza assoluta delle serie di Fourier. *Rend. Accad. Naz. Lincei (6) 2* (1925), pp. 145-149; see also *Opera scelte*, vol. IV, Cremonese, Rome, 1963, pp. 11-16.

[31] WARASZKIEWICZ, Z. Remarque sur un théorème de M. Zygmund. *Pol. Akad. Umiej. Krakow Wydz. Mat.-Przy. A.* (1929), pp. 275-279.

[32] WEYL, H. Inequalities between the two kinds of eigenvalues of a linear transformation. *Proc. Nat. Acad. Sci. U.S.A. 35* (1949), pp. 408-411; see also *Gesammelte abhandlungen*, vol. IV, Springer Verlag, Berlin, 1968, pp. 390-393. MR 11, 37.

[33] YOUNG, W. H. On the multiplication of successions of Fourier constants. *Proc. Roy. Soc. A, 87* (1912), pp. 331-339.

[34] —— On the determination of the summability of a function by means of its Fourier constants. *Proc. London Math. Soc. (2) 12* (1913), pp. 71-88.

[35] ZYGMUND, A. Remarque sur la convergence absolue des séries de Fourier. *J. London Math. Soc. 3* (1928), pp. 194-196.

[36] —— Some points in the theory of trigonometric and power series. *Trans. Amer. Math. Soc. 36* (1934), pp. 586-617.

[37] —— *Trigonometric series.* vols. I, II, University Press, Cambridge, 1968.

(Reçu le 15 janvier 1976)

James Alan Cochran

Department of Mathematics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061