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$$\{(1 + |x|)^{-\alpha}\}_r(\lambda) \leq c(1 + \lambda)^{-\alpha/n}, \lambda > 0.$$

Combining estimates, we obtain

$$\int_{R^n} \phi^p w_1 dx \leq c \int_0^\infty \lambda^{-p} (1 + \lambda)^{-\alpha/n} d\lambda < +\infty$$

if $1 - \frac{\alpha}{n} < p < 1$, as desired. This completes the proof of (ii).

To prove Theorem 3, let $f \in L_w^1$ and $w \in A_1$. Then (11) holds for F , p and w_1 as in the proof of Theorem 1 (ii). (The proof of (11) does not require $R_j f \in L_w^1$.) Hence, by Lemma 2 (see (8)),

$$N(F)(x) \leq c(|F(x, 0)|^{\frac{n-1}{n}})^{*}_{n-1}.$$

Since $F(x, 0) = (f(x), (R_1 f)(x), \dots, (R_n f)(x))$, the conclusion of Theorem 3 follows immediately with $\mu = (n-1)/n$.

To prove the fact stated at the end of the introduction, let

$$f, R_1 f, \dots, R_n f \in L^1.$$

Clearly,

$$P(R_j f)^\wedge(x, t) = \hat{P}(x, t)(R_j f)^\wedge(x) = e^{-2\pi t|x|}(R_j f)^\wedge(x),$$

$$(Q_j f)^\wedge(x, t) = \hat{Q}_j(x, t)f^\wedge(x) = i \frac{x_j}{|x|} e^{-2\pi t|x|} f^\wedge(x) \text{ a.e.},$$

where the Fourier transform is taken in the x variable with t fixed. (Note that for fixed t , $P(x, t)$ belongs to L^1 and $Q_j(x, t)$ belongs to L^2 .) However, these expressions are all equal everywhere since $P(R_j f) = Q_j f$ by Theorem 1 and $P(R_j f) \in L^1$. Therefore, $(R_j f)^\wedge(x) = ix_j |x|^{-1} f^\wedge(x)$, as claimed.

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