Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 22 (1976)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: CLASSICAL THEOREMS ON QUADRATIC RESIDUES
Autor: Berndt, Bruce C.
Bibliographie
DOI: https://doi.org/10.5169/seals-48188

For $p \equiv 7 \pmod 8$, the largest value for S_{21} is 259. The smallest value not assumed is 163. There are several other values between 163 and 259 that are not assumed. The calculations also strongly support the following conjectures. S_{41} and S_{31}, for $p \equiv 1 \pmod 4$; S_{52}, for $p \equiv 3 \pmod 4$; S_{81}, for $p \equiv 1 \pmod 8$; S_{82}, for $p \equiv 7 \pmod 8$; $-S_{84}$, for $p \equiv 5 \pmod 8$; and $S_{12,2}$, for $p \equiv 7 \pmod 8$ and for $p \equiv 11 \pmod{12}$, each assumes all positive, integral values. We refer the reader to the foregoing work here for the translations of these conjectures into conjectures about class numbers.

REFERENCES

[1] APOSTOL, Tom M. Quadratic residues and Bernoulli numbers. *Delta 1* (1968/70) pp. 21-31.

[2] AYOUB, Raymond. *An introduction to the analytic theory of numbers*. American Mathematical Society, Providence, 1963.

[3] ——, S. CHOWLA and H. WALUM. On sums involving quadratic characters. *J. London Math. Soc. 42* (1967), pp. 152-154.

[4] BARRUCAND, Pierre and Harvey COHN. Note on primes of type x^2+32y^2, class number, and residuacity. *J. Reine Angew. Math. 238* (1969), pp. 67-70.

[5] BERGER, A. Sur une sommation de quelques séries. *Nova Acta Regiae Soc. Sci. Upsaliensis 12* (1884), 31 pp.

[6] BERNDT, Bruce C. Character analogues of the Poisson and Euler-Maclaurin summation formulas with applications. *J. Number Theory 7* (1975), pp. 413-445.

[7] —— Periodic Bernoulli numbers, summation formulas and applications. *Theory and application of special functions*. Richard A. Askey, ed., Academic Press, New York, 1975, pp. 143-189.

[8] —— and S. CHOWLA. Zero sums of the Legendre symbol. *Nordisk Mat. Tidskr. 22* (1974), pp. 5-8.

[9] —— and Lowell SCHOENFELD. Periodic analogues of the Euler-Maclaurin and Poisson summation formulas with applications to number theory. *Acta Arith. 28* (1975), pp. 23-68.

[10] BROWN, Ezra. The class number of $Q(\sqrt{-p})$, for $p \equiv 1 \pmod 8$ a prime. *Proc. Amer. Math. Soc. 31* (1972), pp. 381-383.

[11] —— The power of 2 dividing the class-number of a binary quadratic discriminant. *J. Number Theory 5* (1973), pp. 413-419.

[12] —— Class numbers of complex quadratic fields. *J. Number Theory 6* (1974), pp. 185-191.

[13] —— A lemma of Stark. *J. Reine Angew. Math. 265* (1974), p. 26.

[14] —— Class numbers of quadratic fields. *Symp. Mat. 15* (1975), pp. 403-411.

[15] —— and Charles J. PERRY. Class numbers of imaginary quadratic fields having exactly three discriminant divisors. *J. Reine Angew. Math. 260* (1973), pp. 31-34.

[16] CARLITZ, L. Some sums connected with quadratic residues. *Proc. Amer. Math. Soc. 4* (1953), pp. 12-15.

[17] CAUCHY, A. L. *Note XII, Œuvres (1), Tome III*. Gauthier-Villars, Paris, 1882, pp. 359-390.

[18] CHOWLA, S. On a problem of analytic number theory. *Proc. Nat. Inst. Sci. India 13* (1947), pp. 231-232.

[19] CHUNG, Kai-Lai. Note on a theorem on quadratic residues. *Bull. Amer. Math. Soc. 47* (1941), pp. 514-516.

[20] DAVENPORT, Harold. *Multiplicative number theory.* Markham, Chicago, 1967.

[21] DEDEKIND, R. Bemerkungen zur Abhandlung, K. F. Gauss, De nexu inter multitudinem classium, in quas formae binariae secundi gradus distribuunter, earumque determinantem. *Gauss, K. F. Werke, Zweiter Band.* K. Gesell. Wiss., Göttingen, pp. 292-303.

[22] DICKSON, L. E. *History of the theory of numbers, vol. III.* G. E. Stechert and Co., New York, 1934.

[23] DIRICHLET, G. L. Recherches sur diverses applications de l'analyse infinitésimale à la théorie des nombres. *J. Reine Angew. Math. 19* (1839), pp. 324-369.

[24] —— Recherches sur diverses applications de l'analyse infinitésimale à la théorie des nombres, seconde partie. *J. Reine Angew. Math. 21* (1840), pp. 134-155.

[25] FINE, N. J. On a question of Ayoub, Chowla and Walum concerning character sums. *Illinois J. Math. 14* (1970), pp. 88-90.

[26] GAUSS, K. F. De nexu inter multitudinem classium, in quas formae binariae secundi gradus distribuunter, earumque determinantem. *Werke, Zweiter Band.* K. Gesell. Wiss., Göttingen, 1876, pp. 269-291.

[27] GLAISHER, J. W. L. Formulae derived from Gauss's sums, with applications to the series connected with the number or classes of binary forms. *Quart. J. Math. 33* (1901), pp. 289-330.

[28] —— On the distribution of the numbers for which $(\frac{s}{p}) = 1$, or -1, in the octants, quadrants, etc., of *P. Quart J. Math. 34* (1903), pp. 1-27.

[29] —— On the expression for the number of classes of a negative determinant, and on the numbers of positives in the octants of *P. Quart. J. Math. 34* (1903), pp. 178-204.

[30] GUINAND, A. P. On Poisson's summation formula. *Ann. of Math. 42* (1941), pp. 591-603.

[31] HASSE, Helmut. *Vorlesungen über Zahlentheorie.* Springer-Verlag, Berlin, 1964.

[32] —— Über die Klassenzahl des Körpers $P(\sqrt{-2p})$ mit einer Primzahl $p \neq 2$. *J. Number Theory 1* (1969), pp. 231-234.

[33] —— Über die Klassenzahl des Körpers $P(\sqrt{-p})$ mit einer Primzahl $p \equiv 1 \bmod. 2^3$. *Aequa. Math. 3* (1969), pp. 165-169.

[34] —— Über die Teilbarkeit durch 2^3 der Klassenzahl imaginär-quadratischer Zahlkörper mit genau zwei vershiedenen Diskriminanten-primteilern. *J. Reine Angew. Math. 241* (1970), pp. 1-6.

[35] —— Über die Teilbarkeit durch 2^3 der Klassenzahl der quadratischen Zahlkörper mit genau zwei verschiedenen Diskriminanten-primteilern. *Math. Nachr. 46* (1970), pp. 61-70.

[36] HOLDEN, H. On various expressions for h, the number of properly primitive classes for a determinant $-p$, where p is of the form $4n+3$, and is a prime or the product of different primes (second paper). *Mess. Math. 35* (1905/06), pp. 102-110.

[37] —— On various expressions for h, the number of properly primitive classes for any negative determinant, not involving a square factor (third papier). *Mess. Math. 35* (1905/06), pp. 110-117.

[38] —— On various expressions for h, the number of properly primitive classes for a determinant $-p$, where p is of the form $4n+3$, and is a prime or the product of different primes (addition to the second paper). *Mess. Math. 36* (1906/07), pp. 75-77.

[39] —— On various expressions for h, the number of properly primitive classes for a negative determinant not containing a square factor (fifth paper). *Mess. Math. 36* (1906/07), pp. 126-134.

[40] HURWITZ, A. Über die Anzahl der Classen binärer quadratischer Formen von negativer Determinante, *Acta Math. 19* (1895), pp. 351-384.
[41] JOHNSON, Wells and Kevin J. MITCHELL. Symmetries for sums of the Legendre symbol. *Acta Arith. (to appear)*.
[42] KARPINSKI, L. Über die Verteilung der quadratischen Reste. *J. Reine Angew. Math. 127* (1904), pp. 1-19.
[43] LANDAU, E. *Vorlesungen über Zahlentheorie, Band 1*. S. Hirzel, Leipzig, 1927.
[44] LERCH, M. Essais sur le calcul du nombre des classes de formes quadratiques binaires aux coefficients entiers. *Acta Math. 29* (1905), pp. 333-424.
[45] —— Essais sur le calcul du nombre des classes de formes quadratiques binaires aux coefficients entiers. *Acta Math. 30* (1906), pp. 203-293.
[46] MORDELL, L. J. Some applications of Fourier series in the analytic theory of numbers. *Proc. Cambridge Philos. Soc. 24* (1928), pp. 585-596.
[47] MOSER, Leo. A theorem on quadratic residues. *Proc. Amer. Math. Soc. 2* (1951), pp. 503-504.
[48] —— *An introduction to the theory of numbers* (mimeographed lecture notes). Canadian Mathematical Congress, University of Alberta, 1957.
[49] NEVILLE, E. H. A trigonometrical inequality. *Proc. Cambridge Philos. Soc. 47* (1951), pp. 629-632.
[50] OSBORN, G. Some properties of the quadratic residues of primes. *Mess. Math. 25* (1896), pp. 45-47.
[51] PEPIN, P. Nombre des classes de formes quadratiques pour un déterminant donné. *Ann. Sci. d'Ecole Norm. Sup. (2) 3* (1874), pp. 165-208.
[52] PIZER, Arnold K. Type numbers of Eichler orders. *J. Reine Angew. Math. 264* (1973), pp. 76-102.
[53] —— On the 2-part of the class number of imaginary quadratic number fields. *J. Number Theory 8* (1976), pp. 184-192.
[54] PLANCHEREL, Michel. *Sur les congruences (mod 2^m) relatives au nombre des classes des formes quadratiques binaires aux coefficients entiers et a discriminant negatif.* Thèse, Paris, 1908.
[55] PUMPLÜN, Dieter. Über die Klassenzahl imaginär-quadratischer Zahlkörper. *J. Reine Angew. Math. 218* (1965), pp. 23-30.
[56] RÉDEI, L. Über die Klassenzahl des imaginären quadratischen Zahlkörpers. *J. Reine Angew. Math. 159* (1928), pp. 210-219.
[57] —— Über die Wertverteilung des Jacobischen Symbols. *Acta Sci. Math. (Szeged) 13* (1949/50), pp. 242-246.
[58] —— and H. REICHARDT. Die Anzahl der durch 4 teilbaren Invarianten der Klassengruppe eines beliebigen quadratischen Zahlkörpers. *J. Reine Angew. Math. 170* (1934), pp. 69-74.
[59] STARK, H. M. A complete determination of the complex quadratic fields of class-number one. *Mich. Math. J. 14* (1967), pp. 1-27.
[60] WHITEMAN, A. L. Theorems on quadratic residues. *Math. Mag. 23* (1949/50), pp. 71-74.
[61] WOLKE, Dieter. Eine Bemerkung über das Legendre-Symbol. *Monat. Math. 77* (1973), pp. 267-275.
[62] YAMAMOTO, Y. unpublished manuscript.

(Reçu le 20 mai 1976)

Bruce C. BERNDT

Department of Mathematics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801