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For $p \equiv 7 \pmod{8}$, the largest value for S_{21} is 259. The smallest value not assumed is 163. There are several other values between 163 and 259 that are not assumed. The calculations also strongly support the following conjectures. S_{41} and S_{31} , for $p \equiv 1 \pmod{4}$; S_{52} , for $p \equiv 3 \pmod{4}$; S_{81} , for $p \equiv 1 \pmod{8}$; S_{82} , for $p \equiv 7 \pmod{8}$; $-S_{84}$, for $p \equiv 5 \pmod{8}$; and $S_{12,2}$, for $p \equiv 7 \pmod{8}$ and for $p \equiv 11 \pmod{12}$, each assumes all positive, integral values. We refer the reader to the foregoing work here for the translations of these conjectures into conjectures about class numbers.

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