

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 21 (1975)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: CHANGES OF SIGN OF $\text{te}(x) - \text{li } x$
Autor: Diamond, Harold G.
Kapitel: 1. Introduction
DOI: <https://doi.org/10.5169/seals-47326>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 17.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

CHANGES OF SIGN OF $\pi(x) - \text{li } x$

by Harold G. DIAMOND ¹

1. INTRODUCTION

The prime number theorem asserts that $\pi(x)$, the number of primes not exceeding x , is asymptotic to

$$\text{li } x \stackrel{\text{def}}{=} \lim_{\varepsilon \rightarrow 0+} \left(\int_0^{1-\varepsilon} + \int_{1+\varepsilon}^x \right) \frac{du}{\log u}$$

as $x \rightarrow \infty$. It has been shown [12, p. 72] that $\pi(x) < \text{li } x$ for $3/2 \leq x \leq 10^8$, and it was once conjectured that this inequality prevailed for all $x \geq 3/2$. However, this conjecture was disproved by Littlewood [11] who established

THEOREM 1. $\pi(x) - \text{li } x$ changes sign infinitely often as $x \rightarrow \infty$.

Littlewood's proof was simplified by Ingham [5]. In the present article we make a further simplification by eliminating use of the so called explicit formula for ψ (cf. [4], pp. 76-80). The deepest fact which we require from analytic number theory is an estimate of the size of $N(T)$, the number of zeros ρ of the Riemann zeta function satisfying $0 < \text{Im } \rho \leq T$.

The key step in the argument is Theorem 2, which is given in the next section. This result, which is based on another article of Ingham [6], enables us to relate a certain average of the function π to zeros of the Riemann zeta function.

2. A TAUBERIAN THEOREM

We begin by giving an extension of the Wiener-Ikehara tauberian theorem. Our result admits poles and certain "lesser" singularities on the abscissa of convergence of the transformed function. We adhere to the curious convention of expressing the complex variable s as $\sigma + it$.

¹) Research supported in part by a grant from the National Science Foundation.