Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 20 (1974)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON DIOPHANTINE EQUATIONS OF THE FORM $x^2 + D = p^k$
Autor: Cohen, Edward L.
Kapitel: 2. Historical background: D = 7, p = 2 and D = 11, p = 3
DOI: https://doi.org/10.5169/seals-46907

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 28.11.2025

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

2. Historical background: $D = 7, p = 2$ and $D = 11, p = 3$

Ramanujan in 1913 [12], [13] asked whether there were other solutions to the diophantine equation

$$x^2 + 7 = 2^k \tag{2.1}$$

besides the known ones, namely when $k = 3, 4, 5, 7, 15$. This problem was again posed by Ljunggren [7] in 1943, and it was finally shown by Nagell [10] that the k above were the only five solutions. Nagell's paper was written in Norwegian and few knew about its existence although he had posed it shortly afterwards as an exercise in his book [11] on elementary number theory. Hence, thereafter, there were a large number of papers proving the same result (see [5] for an up-to-date list).

It is also interesting to note that equation (2.1) has interesting applications to binary error-correcting codes [3], [14].

Equation (2.1) is of the form

$$x^2 + D = [(D+1)/4]^k . \tag{2.2}$$

So the next equation of interest might seem to be

$$x^2 + 11 = 3^k . \tag{2.3}$$

This equation was solved by Ljunggren and the author [5]. Its only solution occurs when $k = 3$.

3. The main theorem

Theorem 3.1. Let $D \equiv 3 \pmod 8$. Let $p = (D+1)/4$ be a prime $\geqslant 5$. Then the diophantine equation

$$x^2 + D = p^k \tag{3.1}$$

has no solutions.

The remainder of this section will be devoted to a proof of this theorem, a sketch of which was presented in [4]. The last section will be devoted to corollaries and other related results.

Lemma 3.2. There are no solutions to the equation (3.1) when k is even.

Proof. When k is even, the equation can be written as

$$(p^{k/2})^2 - x^2 = D . \tag{3.2}$$

Therefore,