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is a close relationship between the two coefficient algebras. A new proof of this result was given by Solomon [22, Theorem 3]. This proof can be found in [11, p. 475-479).

The theorem was further refined by Brauer [7] and Witt [23]. They showed that for some quotient group $\mathfrak{C}$ of $\mathfrak{B}$, the quotient $\mathfrak{F} = \mathfrak{A}\,\mathfrak{C}$ of $\mathfrak{G}$ has a coefficient algebra of index p^a. The group $\mathfrak{C}$ contains a cyclic normal subgroup $\mathfrak{Z}$ for which $\mathfrak{C}/\mathfrak{Z}$ is Abelian. A proof of this is given in [16, Lemma 1]. Yamada [24], [25] and [26] has investigated coefficient division algebras for certain special types of the groups just described.

The index m of a coefficient algebra must divide the order g of the group. Another bound on the index states that for a prime divisor p of m, the highest power of p dividing m must also divide $q - 1$ for some prime divisor q of g. An exception to this can occur if g is a power of two; we may have $m = 2$ in this case. This theorem is implicit in the work of Witt [23, Satz 12, p. 245] and was stated and proved independently by the author in [16].

Suppose a field **F** is given and the question is asked: which division algebras with center **F** appear as the coefficient algebras in some group algebra. This question has been answered for different fields by several authors in [3], [4], [13], [14], [15], [19] and [27]. Closely related problems have been investigated in [12].

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