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on the Borel sets of R^n. Since for any set $A \subset R^n$, there exists a G_δ set $B \supset A$ with $L(B) = L(A)$, it follows that

$$L(A) \leqslant \Phi_{\mathscr{G},M}(A) \leqslant \Phi_{\mathscr{G},M}(B) = L(B) = L(A),$$
$$\Phi_{\mathscr{G},M}(A) = L(A).$$

Having proved $\Phi_{\mathscr{G},M} = L$, we apply Theorems 2 and 3 to conclude

$$L \leqq \varphi_{\mathscr{F},M} \leqq \Phi_{\mathscr{F},M} \leqq \Phi_{\mathscr{G},M} = L,$$

and the proof is complete.

Corollary 1. [1, p. 163]. Hausdorff n-dimensional sphere outer measure in R^n agrees with L on all the subsets of R^n.

Proof. Recall that Hausdorff n-dimensional sphere outer measure is the outer measure $\Phi_{\mathscr{F},L}$ where $\mathscr{F}$ is the set of all spheres in R^n. Application of Theorem 5 is immediate.

Corollary 2. [5]. Hausdorff n-measure in R^n agrees with L on all subsets of R^n.

Proof. Recall that Hausdorff n-measure is the outer measure $\Phi_{\mathscr{F},M}$ where $\mathscr{F}$ is the set of all subsets of R^n and $M(A) = (n!)^{-1}\,\Gamma(\frac{1}{2})^{n-1}\,\Gamma\left(\frac{n+1}{2}\right)(\text{diam } A)^n$. In Theorem 5, take α to be an n-sphere. The isodiametric inequality implies that $M \geqslant L$ on $\mathscr{F}$ and $M = L$ on $\mathscr{G}$. Application of Theorem 5 completes the proof.

Corollary 3. L is invariant under rotation.

Proof. For a fixed rotation, merely take $\mathscr{F}$ to be the set of rotated n-cubes and $M = L$.

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