infinitely many integer solutions can always be found if V is linear, i.e. is a subspace. For the non-linear case we have neither a good generalization of Dirichlet's Theorem nor anything like Roth's Theorem.

Suppose now that V is a hypersurface containing no integer point $\mathbf{x} \neq \mathbf{0}$ and defined by the equation $F(\mathbf{x}) = 0$ where F is a form of degree d with rational integer coefficients. For every integer point $\mathbf{x} \neq \mathbf{0}$ we have $| F(\mathbf{x}) | \geqq 1$, and since $\left| \frac{\partial}{\partial x_i} F(\mathbf{x}) \right| \leqq c_1 | \mathbf{x} |^{d-1}$ $(i=1, ..., n)$, the distance from $\mathbf{x}$ to V is $\geqq c_2 | \mathbf{x} |^{1-d}$, which in turn implies that

$$\psi(V, \mathbf{x}) \geqq c_3 | \mathbf{x} |^{-d},$$

where the constants depend only on V. This inequality may be interpreted as a generalization of Liouville's Theorem. Any improvement of this inequality, even though perhaps it may apply only to special classes of non-linear hypersurfaces, would be of great interest and would shed light on certain diophantine equations different from the equations with norm forms discussed in §10.

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