

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 17 (1971)  
**Heft:** 1: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** ON THE CONCEPT OF DIRECTED DISTANCE  
**Autor:** Singmaster, David  
**Bibliographie**  
**DOI:** <https://doi.org/10.5169/seals-44574>

#### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

#### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

#### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

**Download PDF:** 21.05.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

Further, Axiom,  $i=1, 3, 4, 5$ , holds in  $(X, e)$  if it holds in  $(X, d)$ . If Axiom 2 holds in  $(X, d)$  and  $f$  is one to one, the Axiom 2 holds in  $(X, e)$ .

*Corollary 17.* If  $(X, d)$  is a pseudo-directed distance space, then so is  $(X, e)$ . If  $(X, d)$  is a directed distance space and  $f$  is one to one, then  $(X, e)$  is also a directed distance space.

*Remarks.* In retrospect, I notice that the associative law of the group  $G$  has never been used. Hence, the role of  $G$  could be played by a somewhat weaker concept, namely that of a loop with the inverse property. [1, p. 7].

I am indebted to David Makinson for the idea of Proposition 5.

Since first writing this paper, it has been pointed out to me that the axioms for a directed distance space are similar to the axioms for an affine space. [2, p. 420-5, esp. (6) and (7) on p. 421. There is a misprint in (6).] There, the values are assumed to lie in a vector space rather than in a group. In addition to axioms 1, 2 and 4, it is assumed in [2] that

$$6. \forall x \in X, \forall g \in G, \exists ! y \in X : d(y, x) = g.$$

(In the notation of [2], this would be written as  $g + x = y$ ). This assumption is equivalent to the assertion that for each  $a \in X$ , the mapping  $f_a$  of Theorem 14 is onto. Hence our result, Corollary 15, could be improved to assert that  $f_a$  is an isometry onto and we can say that the only  $G$ -directed distance space satisfying 6 is  $G$  itself.

#### REFERENCES

1. HALL, M. Jr.: *The Theory of Groups*, Macmillan, New York, 1959.
2. MACLANE, S. and G. BIRKHOFF: *Algebra*, Macmillan, New York, 1967.

(*Reçu le 16 février 1971*)

Polytechnic of the South Bank,  
London S.E.1