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$$x_3 = \tfrac{1}{2}\frac{\delta^1}{1} + \tfrac{1}{4}\frac{\delta^2}{2} + \tfrac{1}{4}\frac{\delta^3}{3}$$

. . .

$$x_i = \tfrac{1}{2}\frac{\delta^1}{1} + \tfrac{1}{4}\frac{\delta^2}{2} + \cdots + \frac{1}{2^{i-1}}\frac{\delta^{i-1}}{i-1} + \frac{1}{2^{i-1}}\frac{\delta^i}{i}$$

. . .

For each i, $x_i \in K(S)$ [1, p. 10]. If

$$x_0 = \lim_{i\to\infty} x_i = \lim_{i\to\infty} \Big(\sum_{n=1}^{i-1} \frac{1}{2^n}\frac{\delta^n}{n} + \frac{1}{2^{i-1}}\frac{\delta^i}{i}\Big) = \lim_{i\to\infty} \sum_{n=1}^{i} \frac{1}{2^n}\frac{\delta^n}{n},$$

then $x_0 \notin K(S)$ but $x_0 \in \mathrm{cl}\, K(S)$ where $\mathrm{cl}\, K(S)$ is the closure of $K(S)$ in m. We remark that this shows that in a linear topological space the convex hull of a convex set may not be closed and, hence, may not be compact [2, p. 141]. We note but do not prove that the closed convex hull, $\mathrm{cl}\, K(S)$, of S is compact.

Let L be the set of all linear combinations of all finite subsets of S. Then L with the relative topology of m [2, p. 51] is the smallest linear subspace [1, p. 2] of m containing $K(S)$, and $K'(S) = L \cap \mathrm{cl}\, K(S)$ is the closed convex hull of S in L. To complete our proof it suffices to show that $K'(S)$ is not a compact subset of L.

The sequence $\{x_i\}_{i\in\omega}$ defined above is a closed subset of L since its only cluster point $x_0 \notin L$. For each $k \in \omega$ let $A_k = \{x \in L : x \notin \{x_i\}_{i=k}^{\infty}\}$. Each A_k is an open subset of L and $A = \{A_k : k \in \omega\}$ is an open cover of $K'(S)$. However, if $A_{k(1)}, A_{k(2)}, \ldots, A_{k(n)}$ is a finite subset of A and if j is any integer greater than $\max\{k(1), k(2), \ldots, k(n)\}$, then $x_j \notin U_{i=1}^{n} A_{k(i)}$. Since $x_j \in \{x_i\}_{i\in\omega} \subset K'(S)$ it follows that A does not contain a finite subset which covers $K'(S)$ and, hence, that $K'(S)$ is not a compact subset of L which completes the proof.

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