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$$\begin{aligned}
 x_3 &= \frac{1}{2} \frac{\delta^1}{1} + \frac{1}{4} \frac{\delta^2}{2} + \frac{1}{4} \frac{\delta^3}{3} \\
 &\quad \cdot \quad \cdot \quad \cdot \\
 x_i &= \frac{1}{2} \frac{\delta^1}{1} + \frac{1}{4} \frac{\delta^2}{2} + \cdots + \frac{1}{2^{i-1}} \frac{\delta^{i-1}}{i-1} + \frac{1}{2^{i-1}} \frac{\delta^i}{i} \\
 &\quad \cdot \quad \cdot \quad \cdot
 \end{aligned}$$

For each  $i$ ,  $x_i \in K(S)$  [1, p. 10]. If

$$x_0 = \lim_{i \rightarrow \infty} x_i = \lim_{i \rightarrow \infty} \left( \sum_{n=1}^{i-1} \frac{1}{2^n} \frac{\delta^n}{n} + \frac{1}{2^{i-1}} \frac{\delta^i}{i} \right) = \lim_{i \rightarrow \infty} \sum_{n=1}^i \frac{1}{2^n} \frac{\delta^n}{n},$$

then  $x_0 \notin K(S)$  but  $x_0 \in \text{cl } K(S)$  where  $\text{cl } K(S)$  is the closure of  $K(S)$  in  $m$ . We remark that this shows that in a linear topological space the convex hull of a convex set may not be closed and, hence, may not be compact [2, p. 141]. We note but do not prove that the closed convex hull,  $\text{cl } K(S)$ , of  $S$  is compact.

Let  $L$  be the set of all linear combinations of all finite subsets of  $S$ . Then  $L$  with the relative topology of  $m$  [2, p. 51] is the smallest linear subspace [1, p. 2] of  $m$  containing  $K(S)$ , and  $K'(S) = L \cap \text{cl } K(S)$  is the closed convex hull of  $S$  in  $L$ . To complete our proof it suffices to show that  $K'(S)$  is not a compact subset of  $L$ .

The sequence  $\{x_i\}_{i \in \omega}$  defined above is a closed subset of  $L$  since its only cluster point  $x_0 \notin L$ . For each  $k \in \omega$  let  $A_k = \{x \in L : x \notin \{x_i\}_{i=k}^\infty\}$ . Each  $A_k$  is an open subset of  $L$  and  $A = \{A_k : k \in \omega\}$  is an open cover of  $K'(S)$ . However, if  $A_{k(1)}, A_{k(2)}, \dots, A_{k(n)}$  is a finite subset of  $A$  and if  $j$  is any integer greater than  $\max\{k(1), k(2), \dots, k(n)\}$ , then  $x_j \notin \bigcup_{i=1}^n A_{k(i)}$ . Since  $x_j \in \{x_i\}_{i \in \omega} \subset K'(S)$  it follows that  $A$  does not contain a finite subset which covers  $K'(S)$  and, hence, that  $K'(S)$  is not a compact subset of  $L$  which completes the proof.

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