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surely obvious what I mean by the disadvantage, and it is obviously bad; avoid it. The advantage deserves further emphasis.

The writer must anticipate and avoid the reader's difficulties. As he writes, he must keep trying to imagine what in the words being written may tend to mislead the reader, and what will set him right. I'll give examples of one or two things of this kind later; for now I emphasize that keeping a specific reader in mind is not only helpful in this aspect of the writer's work, it is essential.

Perhaps it needn't be said, but it won't hurt to say, that the audience actually reached may differ greatly from the intended one. There is nothing that guarantees that a writer's aim is always perfect. I still say it's better to have a definite aim and hit something else, than to have an aim that is too inclusive or too vaguely specified and have no chance of hitting anything. Get ready, aim, and fire, and hope that you'll hit a target: the target you were aiming at, for choice, but some target in preference to none.

## 4. Organize first

The main contribution that an expository writer can make is to organize and arrange the material so as to minimize the resistance and maximize the insight of the reader and keep him on the track with no unintended distractions. What, after all, are the advantages of a book over a stack of reprints? Answer: efficient and pleasant arrangement, emphasis where emphasis is needed, the indication of interconnections, and the description of the examples and counterexamples on which the theory is based; in one word, organization.

The discoverer of an idea, who may of course be the same as its expositor, stumbled on it helter-skelter, inefficiently, almost at random. If there were no way to trim, to consolidate, and to rearrange the discovery, every student would have to recapitulate it, there would be no advantage to be gained from standing "on the shoulders of giants", and there would never be time to learn something new that the previous generation did not know.

Once you know what you want to say, and to whom you want to say it, the next step is to make an outline. In my experience that is usually impossible. The ideal is to make an outline in which every preliminary heuristic discussion, every lemma, every theorem, every corollary, every remark, and every proof are mentioned, and in which all these pieces occur in an

order that is both logically correct and psychologically digestible. In the ideal organization there is a place for everything and everything is in its place. The reader's attention is held because he was told early what to expect, and, at the same time and in apparent contradiction, pleasant surprises keep happening that could not have been predicted from the bare bones of the definitions. The parts fit, and they fit snugly. The lemmas are there when they are needed, and the interconnections of the theorems are visible; and the outline tells you where all this belongs.

I make a small distinction, perhaps an unnecessary one, between organization and arrangement. To organize a subject means to decide what the main headings and subheadings are, what goes under each, and what are the connections among them. A diagram of the organization is a graph, very likely a tree, but almost certainly not a chain. There are many ways to organize most subjects, and usually there are many ways to arrange the results of each method of organization in a linear order. The organization is more important than the arrangement, but the latter frequently has psychological value.

One of the most appreciated compliments I paid an author came from a fiasco; I botched a course of lectures based on his book. The way it started was that there was a section of the book that I didn't like, and I skipped it. Three sections later I needed a small fragment from the end of the omitted section, but it was easy to give a different proof. The same sort of thing happened a couple of times more, but each time a little ingenuity and an ad hoc concept or two patched the leak. In the next chapter, however, something else arose in which what was needed was not a part of the omitted section but the fact that the results of that section were applicable to two apparently very different situations. That was almost impossible to patch up, and after that chaos rapidly set in. The organization of the book was tight; things were there because they were needed; the presentation had the kind of coherence which makes for ease in reading and understanding. At the same time the wires that were holding it all together were not obtrusive; they became visible only when a part of the structure was tampered with.

Even the least organized authors make a coarse and perhaps unwritten outline; the subject itself is, after all, a one-concept outline of the book. If you know that you are writing about measure theory, then you have a two-word outline, and that's something. A tentative chapter outline is something better. It might go like this: I'll tell them about sets, and then measures, and then functions, and then integrals. At this stage you'll want to make some decisions, which, however, may have to be rescinded later;

you may for instance decide to leave probability out, but put Haar measure in.

There is a sense in which the preparation of an outline can take years, or, at the very least, many weeks. For me there is usually a long time between the first joyful moment when I conceive the idea of writing a book and the first painful moment when I sit down and begin to do so. In the interim, while I continue my daily bread and butter work, I daydream about the new project, and, as ideas occur to me about it, I jot them down on loose slips of paper and put them helter-skelter in a folder. An "idea" in this sense may be a field of mathematics I feel should be included, or it may be an item of notation; it may be a proof, it may be an aptly descriptive word, or it may be a witticism that, I hope, will not fall flat but will enliven, emphasize, and exemplify what I want to say. When the painful moment finally arrives, I have the folder at least; playing solitaire with slips of paper can be a big help in preparing the outline.

In the organization of a piece of writing, the question of what to put in is hardly more important than what to leave out; too much detail can be as discouraging as none. The last dotting of the last i, in the manner of the old-fashioned Cours d'Analyse in general and Bourbaki in particular, gives satisfaction to the author who understands it anyway and to the helplessly weak student who never will; for most serious-minded readers it is worse than useless. The heart of mathematics consists of concrete examples and concrete problems. Big general theories are usually afterthoughts based on small but profound insights; the insights themselves come from concrete special cases. The moral is that it's best to organize your work around the central, crucial examples and counterexamples. The observation that a proof proves something a little more general than it was invented for can frequently be left to the reader. Where the reader needs experienced guidance is in the discovery of the things the proof does not prove; what are the appropriate counterexamples and where do we go from here?

# 5. THINK ABOUT THE ALPHABET

Once you have some kind of plan of organization, an outline, which may not be a fine one but is the best you can do, you are almost ready to start writing. The only other thing I would recommend that you do first is to invest an hour or two of thought in the alphabet; you'll find it saves many headaches later.