

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 16 (1970)  
**Heft:** 1: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** NAIVELY CONSTRUCTIVE APPROACH TO BOUNDEDNESS PRINCIPLES, WITH APPLICATIONS TO HARMONIC ANALYSIS  
**Autor:** Edwards, R. E. / Price, J. F.  
**Anhang:** Appendix Rudin-Shapiro sequences  
**DOI:** <https://doi.org/10.5169/seals-43866>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 14.03.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

are then seen from (10.9), (10.10) and (10.11) to satisfy

$$\left. \begin{aligned} & \| Q'_j \| \leq 1, \\ & \text{sp } (Q'_j) \subseteq [A_j], \\ & \left| \int v D_{\Delta_j} Q'_j d\lambda_G \right| \geq (1 - 3v^{-\frac{1}{2}}) \| D_{\Delta_j} \|_1 \end{aligned} \right\} \quad (10.13)$$

provided  $v$  is chosen  $\geq 9 \| D_{\Delta_j} \|_1^{-1}$ . In view of (7.6), we may choose the integer  $v \geq \max_j (36, 9 \| D_{\Delta_j} \|_1^{-1})$ . Then (10.13) shows that there are unimodular complex numbers  $\xi_j$  such that the  $Q_j = \xi_j Q'_j$  satisfy (7.7).

## APPENDIX

### *Rudin-Shapiro sequences*

**A.1 NOTATIONS AND DEFINITIONS.** As hitherto, all topological groups  $G$  are assumed to be Hausdorff; and, for any locally compact group  $G$ ,  $\lambda_G$  will denote a selected left Haar measure, with respect to which the Lebesgue spaces  $L^p(G)$  are to be formed.  $C_c(G)$  denotes the set of complex-valued continuous functions on  $G$  having compact supports.

If  $X$  and  $Y$  are topological groups,  $\text{Hom}(X, Y)$  denotes the set of continuous homomorphisms of  $X$  into  $Y$ .

Suppose henceforth  $G$  to be locally compact. As in 5.1, if  $k \in C_c(G)$ ,  $T_k$  will denote the convolution operator

$$f \mapsto f * k$$

with domain  $C_c(G)$  and range in  $C_c(G)$ ; and  $\| k \|_{p,q}$  will denote the  $(p, q)$ -norm of this operator, i.e., the smallest real number  $m \geq 0$  such that

$$\| f * k \|_q \leq m \| f \|_p \quad (f \in C_c(G)).$$

It is well-known that, if  $G$  is Abelian,  $\| k \|_{2,2}$  is equal to

$$\| \hat{k} \|_\infty = \sup_{\gamma \in \Gamma} | \hat{k}(\gamma) |,$$

where  $\Gamma$  is the character group of  $G$  and  $\hat{k}$  is the Fourier transform of  $k$ . (Something similar is true whenever  $G$  is compact, but we shall not use this.)

$U$ -RS-sequences on  $G$  are as defined in 5.4.

In A.2-A.4 we are concerned with conditions on  $G$  sufficient to ensure the possibility of constructing  $U$ -RS-sequences on  $G$  for certain choices of  $U$ . In A.5 we use Rudin-Shapiro sequences on infinite compact Abelian groups to support statements made in 7.5.

A.2 THE ABELIAN CASE. If  $G$  is Abelian and nondiscrete, the methods of § 2 of [5] show how to construct (reasonably explicitly) a  $U$ -RS-sequence  $(h_n)$  on  $G$  for any preassigned nonvoid open  $U \subseteq G$ ; see also [7], (37.19.b). In addition, we may assume that each  $\hat{h}_n$  is integrable on  $\Gamma$ , the character group of  $G$ . [To see this, let  $V$  be a compact neighbourhood of the origin of  $G$  and let  $W$  be a nonvoid subset of  $U$  such that  $V + W \subseteq U$ . Let  $\{u_i\}$  be an approximate identity on  $G$  comprised of functions in  $C_c(G)$  with supports in  $V$  and Fourier transforms in  $L^1(\Gamma)$ . Finally, let  $(k_n)$  be a  $W$ -RS-sequence; then for each  $n \in \mathbb{N}$  we may select  $i_n$  so that  $(k_n * u_{i_n})$  is a  $U$ -RS-sequence with the further property that  $(k_n * u_{i_n})^\wedge = \hat{k}_n \hat{u}_{i_n} \in L^1(\Gamma)$ , as required.] We take this construction for granted (but see A.5 below) and use it to show how to construct  $U$ -RS-sequences on certain non-Abelian groups  $G$ . The basis of the extension is a simple technique of passage from a quotient group to the original, the crucial step being A.3.2 below.

### A.3 THE NOT-NECESSARILY ABELIAN CASE.

A.3.1 Assume here that  $K$  is a compact normal subgroup of  $G$ . Let  $\lambda_K$  be normalised so that  $\lambda_K(K) = 1$ ; and let  $\pi : x \mapsto \bar{x}$  denote the natural mapping of  $G$  onto  $G/K$ .

If  $f \in C_c(G)$ , the function  $f'$  on  $G/K$  defined by

$$f'(\bar{x}) = \int_K f(xt) d\lambda_K(t) \tag{A.1}$$

belongs to  $C_c(G/K)$ ; cf. [7], (15.21). If  $g \in C_c(G/K)$ ,  $g \circ \pi \in C_c(G)$  and

$$(g \circ \pi)' = g. \tag{A.2}$$

If  $\tau_a$  denotes left-translation by amount  $a$ , it is verifiable that  $(\tau_a f)' = \tau_{\bar{a}} f'$ . From this it follows that the disposable factors in  $\lambda_G$  and  $\lambda_{G/K}$  can be mutually adjusted so that

$$\int_G f d\lambda_G = \int_{G/K} f' d\lambda_{G/K} \tag{A.3}$$

for  $f \in C_c(G)$ . Using (A.3), a direct calculation confirms that

$$(f * (k \circ \pi))' = f' * k \tag{A.4}$$

whenever  $f \in C_c(G)$  and  $k \in C_c(G/K)$ .

Another consequence of (A.3) is that for  $1 \leq p \leq \infty$

$$\|f\|_p \geq \|f'\|_p \tag{A.5}$$

for every  $f \in C_c(G)$ ; and that for  $0 < p \leq \infty$

$$\|f\|_p = \|f'\|_p \tag{A.6}$$

for every  $f \in C_c(G;K)$ , the set of  $f \in C_c(G)$  which are constant on cosets modulo  $K$ .

A.3.2 Let  $k \in C_c(G/K)$ . Then

$$\|k \circ \pi\|_{p,q} \leq \|k\|_{p,q} \tag{A.7}$$

PROOF. For  $f \in C_c(G)$ ,  $f * (k \circ \pi) \in C_c(G;K)$  and (A.6) gives

$$\|f * (k \circ \pi)\|_q = \|(f * (k \circ \pi))'\|_q,$$

which by (A.4)

$$\begin{aligned} &= \|f' * k\|_q \\ &\leq \|f'\|_p \|k\|_{p,q} \\ &\leq \|f\|_p \|k\|_{p,q}, \end{aligned}$$

the last step by (A.5). Whence (A.7).

A.3.3 If  $(h_n)$  is a  $V$ -RS-sequence on  $G/K$  and  $U = \pi^{-1}(V)$ , then  $(h_n \circ \pi)$  is a  $U$ -RS-sequence on  $G$ .

PROOF. In view of A.3.2 it suffices to note that

$$\begin{aligned} \text{supp } (h_n \circ \pi) &= \pi^{-1}(\text{supp } h_n) \\ &\subseteq \pi^{-1}(V), \end{aligned}$$

$$\begin{aligned} \|h_n \circ \pi\|_\infty &= \|h_n\|_\infty, \\ \|h_n \circ \pi\|_2 &= \|h_n\|_2, \end{aligned}$$

the last two because of (A.6) and (A.2).

A.3.4 Suppose that  $K$  is a compact normal subgroup of  $G$  and that one can construct  $V$ -RS-sequences on  $G/K$  for any given nonvoid open  $V \subseteq G/K$ . Then one can construct  $U$ -RS-sequences on  $G$  for any given open subset  $U$  of  $G$  which contains  $K$ .

PROOF. Apply A.3.3, taking a nonvoid open subset  $W$  of  $G$  such that  $KW \subseteq U$ , and noting that  $V = \pi(W)$  is then nonvoid and open in  $G/K$  and that  $\pi^{-1}(V) = KW \subseteq U$ .

A.3.5 Let  $\delta(G)$  be the closure in  $G$  of the derived (= commutator) subgroup of  $G$ , and suppose that  $\delta(G)$  is compact and nonopen in  $G$ . Then one can construct  $U$ -RS-sequences on  $G$  for any given open subset  $U$  of  $G$  containing  $\delta(G)$ . (Note that, since  $\delta(G)$  is a closed subgroup of  $G$ , it is nonopen in  $G$  if and only if it has empty interior, or if and only if it is locally null for  $\lambda_G$ .)

PROOF. This follows from A.2 and A.3.4 because:  $\delta(G)$  is in any case a normal subgroup of  $G$  such that  $G/\delta(G)$  is LCA [see [7], (5.22), (5.26), (23.8)]; and  $\delta(G)$  is nonopen in  $G$  if and only if  $G/\delta(G)$  is nondiscrete ([7], (5.21)).

A.3.6 The hypotheses of A.3.5 are satisfied in any one of the following cases (all groups being assumed Hausdorff and locally compact):

(i)  $G = G_1 \times G_2$ , where  $\delta(G_1)$  and  $\delta(G_2)$  are compact and  $\delta(G_1)$  is nonopen in  $G_1$  (hence in particular if  $G = A \times B$ , where  $A$  is nondiscrete Abelian and  $\delta(B)$  is compact);

(ii)  $\delta(G)$  is compact and there exists an open connected subset  $W$  of  $G$  such that  $e \in W \not\subseteq \delta(G)$  (hence in particular if  $G$  is compact and connected and  $\delta(G) \neq G$ );

(iii)  $\delta(G)$  is compact and, for some Abelian  $A$ , some  $\varphi \in \text{Hom}(G, A)$  and some connected open subset  $W$  of  $G$ , we have  $e \in W$  and  $\varphi|_W$  non-constant (hence in particular if  $G$  is compact and connected and  $\text{Hom}(G, A)$  is nontrivial);

(iv)  $G = \varphi(H)$ , where  $\varphi \in \text{Hom}(G, H)$  is such that  $\text{Ker } \varphi$  is locally countable (that is, such that  $\text{Ker } \varphi$  intersects each compact set in a countable set), and where  $\delta(H)$  is compact and nonopen in  $H$ .

PROOF. (i) It is evident that  $\delta(G) \subseteq \delta(G_1) \times \delta(G_2)$ , which shows that  $\delta(G)$  is compact and nonopen in  $G$  [if it were open,  $\delta(G_1) = \text{pr}_{G_1}(\delta(G_1) \times \delta(G_2))$  would have interior points].

(ii) Were  $\delta(G)$  to be open in  $G$ ,  $W$  would be a disjoint union of  $W \cap \delta(G)$  and  $W \cap (G \setminus \delta(G))$ , each relatively open in  $W$ . Since

$e \in W \cap \delta(G)$ , connectedness of  $W$  would imply that  $W \cap (G \setminus \delta(G)) = \emptyset$ , i.e.,  $W \subseteq \delta(G)$ , a contradiction.

(iii)  $\text{Ker } \varphi$  is a closed subgroup of  $G$  containing  $\delta(G)$ ; since  $W \not\subseteq \text{Ker } \varphi$ , it follows that  $W \not\subseteq \delta(G)$ . Now use (ii).

(iv) Clearly,

$$\delta(G) \subseteq \overline{\varphi(\delta(H))} = \varphi(\delta(H))$$

is compact. Suppose  $\delta(G)$  were open in  $G$ . Then  $\varphi(\delta(H))$  has interior points, and the same would be true of

$$\varphi^{-1}(\varphi(\delta(H))) = S\delta(H),$$

where  $S = \text{Ker } \varphi$ . So there would exist a compact neighbourhood  $V$  of the identity in  $H$  such that

$$V \subseteq S\delta(H)$$

and so

$$V = V \cap (S\delta(H)).$$

But, if  $y \in V \cap (S\delta(H))$ ,  $y = sz$  for some  $s \in S$  and  $z \in \delta(H)$ , hence  $s = yz^{-1} \in V\delta(H)^{-1}$ , and so  $s \in (V\delta(H)^{-1}) \cap S$ , which is countable by hypothesis, say  $\{s_n : n \in N\}$ . But then

$$y \in \bigcup_{n \in N} s_n \delta(H).$$

Thus

$$V = V \cap (S\delta(H)) \subseteq \bigcup_{n \in N} s_n \delta(H)$$

and so, since  $\lambda_H(\delta(H)) = 0$ ,

$$0 < \lambda_H(V) \leq \sum_{n \in N} \lambda_H(\delta(H)) = 0,$$

a contradiction.

**A.3.7 REMARKS.** (i) A.3.6 (iii) suffices to show that any finite-dimensional unitary group  $U(n)$  satisfies the hypotheses of A.3.5. [For  $U(n)$  is compact and connected (see [7], (7.15)); and we may apply A.3.6 (iii) with  $A = T$ , the circle group, and  $\varphi = \det$ .]

On the other hand, it is easy to see (cf. A.3.6 (i) and its proof) that if  $G = \prod_{i \in I} G_i$ , where the  $G_i$  are compact and at least one of them satisfies the hypothesis of A.3.5, then  $G$  satisfies the said hypotheses.

So every product of unitary groups satisfies the hypotheses of A.3.5.

(ii) The hypotheses of A.3.5 are also satisfied if  $G = G_1 \circledast G_2$ , the semidirect product of  $G_1$  and  $G_2$  (see [7], (2.6) and (6.20)), provided  $G_1$  is compact and  $\delta(G_2)$  is compact and nonopen in  $G_2$  (hence in particular if  $G = A \circledast B$ , where  $A$  is compact and  $B$  is nondiscrete and Abelian). In fact,  $\delta(G) \subseteq G_1 \times \delta(G_2)$  and the proof proceeds as for A.3.6 (i).

A.4 THE OPERATORS  $f \mapsto k * f$ . Retaining the notations introduced in A.3, it turns out that (cf. (A.4))

$$((k \circ \pi) * f)' = k * f^{\vee\vee} \tag{A.8}$$

for every  $f \in C_c(G)$  and  $k \in C_c(G/K)$ , where, for any function  $g$  with domain a group  $X$ ,  $\check{g}$  denotes the function  $x \mapsto g(x^{-1})$  with domain  $X$ . As a consequence, the results of A.3 have direct analogues for the operator  $f \mapsto k * f$ , provided  $G/K$  is unimodular, which is so if and only if  $G$  is unimodular.

### A.5 CONCERNING 7.5.

A.5.1 Throughout A.5 we suppose  $G$  to be infinite compact Abelian. Let  $\Gamma_0$  be any infinite subsemigroup of the character group  $\Gamma$  of  $G$ ;  $0 \in \Gamma_0$ . The construction described in § 2 of [5] may be employed to produce t.p.s.  $f_n$  ( $n \in N$ ) on  $G$  which, together with their spectra  $S_n$ , satisfy the conditions:

$$\left. \begin{aligned} S_0 &= \{0\}, S_n \subseteq \Gamma_0, |S_n| = 2^n \\ B2^{n/2} &\leq \|f_n\|_s \leq A2^{n/2} \quad (1 \leq s \leq \infty), \\ \|f_n\|_{2,2} &= \|\hat{f}_n\|_\infty \leq 1, \\ \hat{f}_n &= \varphi \text{ on } S_n, 0 \text{ on } \Gamma \setminus S_n, \end{aligned} \right\} \tag{A.9}$$

where  $A$  and  $B$  are positive absolute constants and  $\varphi$  is a function on  $\Gamma$  with  $\text{Ran } \varphi \subseteq \{-1, 0, 1\}$  and  $|\varphi(\gamma)| = 1$  if and only if  $\gamma \in S_n$ . (When  $G = T$ , these  $f_n$  are virtually the original Rudin-Shapiro t.p.s. In the terminology adopted in 5.4 above the  $h_n = 2^{-n/2} f_n$  constitute a  $G$ -RS-sequence on  $G$ .)

If we now choose  $\alpha_n \in \Gamma$  inductively so that, on writing  $F_n = \alpha_n + S_n$ , we have

$$\alpha_{n+1} \in \Gamma_0 \setminus [(F_0 \cup \dots \cup F_n) - S_{n+1}],$$

then

$$\left. \begin{aligned} |F_n| &= |S_n| = 2^n, F_n \subseteq \Gamma_0, \\ F_n \cap F_m &= \emptyset \text{ if } m \neq n, \end{aligned} \right\} \tag{A.10}$$

and the t.p.s

$$w_n = 2^{-n/2} \alpha_n f_n \tag{A.11}$$

satisfy the relations

$$\left. \begin{aligned} & \|w_n\|_\infty \leq A, \hat{w}_n = 2^{-n/2} \varphi_n, \\ & \text{Ran } \varphi_n \subseteq \{-1, 0, 1\}, |\varphi_n(\gamma)| = 1 \text{ if and only if } \gamma \in F_n. \end{aligned} \right\} \tag{A.12}$$

From (A.10) and (A.12) it follows that at least one of the sets  $A_n = \varphi_n^{-1}(\{1\})$ ,  $B_n = \varphi_n^{-1}(\{-1\})$  has not fewer than  $2^{n-1}$  elements. Define  $\varepsilon_n = 1$ ,  $C_n = A_n$  if  $|A_n| \geq 2^{n-1}$  and  $\varepsilon_n = -1$ ,  $C_n = B_n$  if  $|A_n| < 2^{n-1}$ . Then

$$\left. \begin{aligned} & (\varepsilon_n w_n)^\wedge(\gamma) = 2^{-n/2} \text{ if } \gamma \in C_n, \\ & C_n \subseteq F_n, |C_n| \geq 2^{n-1}. \end{aligned} \right\} \tag{A.13}$$

A.5.2 In terms of the construction given in A.5.1, it is possible to write down any number of continuous functions  $f$  on  $G$  and sequences  $(\Delta_j)$  of finite subsets of  $\Gamma_0$  such that

$$\left. \begin{aligned} & \Delta_j \subseteq \Delta_{j+1}, \\ & \text{sp}(f) \subseteq \Gamma_0, \\ & S_{\Delta_j} f(0) \text{ is real and } \lim_{j \rightarrow \infty} S_{\Delta_j} f(0) = \infty, \\ & \sum_{\gamma \in \Gamma} |\hat{f}(\gamma)| = \infty; \end{aligned} \right\} \tag{A.14}$$

cf. the statements made in 7.5.

Indeed, if  $(c_n)_{n=0}^\infty$  is a sequence of real numbers satisfying

$$c_n \geq 0, \sum_{n=0}^\infty c_n < \infty, \sum_{n=0}^\infty 2^{n/2} c_n = \infty, \tag{A.15}$$

and if

$$\Delta_j = C_0 \cup \dots \cup C_j, \tag{A.16}$$

it suffices to take

$$f = \sum_{n=0}^\infty c_n \varepsilon_n w_n, \tag{A.17}$$

(A.14) being then a simple consequence of (A.12) and (A.13).

However, it is a consequence of the choice of the  $\gamma_n$  and  $\alpha_n$  and of (A.12) [on evaluating the Fourier series of  $w_n$  at 0] that  $||A_n| - |B_n|| \leq 2^{n/2}$ , which implies that  $C_n$  contains only about one half the elements of  $F_n$ , so that  $\bigcup_{j=1}^{\infty} \Delta_j$  falls far short of exhausting  $\Gamma_0$ . In particular,  $(\Delta_j)$  is not a convergence grouping of the sort described in § 7.

A.5.3 Two further consequences of the construction in A.5.1 are perhaps worth mentioning in passing.

(i) For any complex-valued sequence  $(c_n)_{n=1}^{\infty}$  such that

$$\sum_{n=1}^{\infty} |c_n| < \infty, \tag{A.18}$$

the formula

$$g = \sum_{n=1}^{\infty} c_n w_n \tag{A.19}$$

yields a continuous function  $g \in C(G)$ . It is easy to specify choices of  $(c_n)$  in accord with (A.18), and of nonnegative functions  $\eta$  on  $\Gamma$  such that

$$\lim_{\gamma \rightarrow \infty} \eta(\gamma) = 0, \tag{A.20}$$

for which

$$\sum_{\gamma \in \Gamma} |\hat{g}(\gamma)|^{2-2\eta(\gamma)} = \infty. \tag{A.21}$$

One might, for example, take  $c_n = n^{-2}$  and  $\eta(\gamma) = 6n^{-1} \log n$  for  $\gamma \in F_n$  ( $n = 1, 2, \dots$ ) and  $\eta(\gamma) = 0$  for  $\gamma \in \Gamma \setminus F$ , where  $F = \bigcup_{n=1}^{\infty} F_n$ .

This is an analogue of a well-known result of Banach for the case  $G = T$ ; it provides numerous reasonably constructive counter-examples to conjectural improvements of the Hausdorff-Young theorem.

(ii) Take  $(c_n)$ ,  $\eta$  and  $g$  as in (i) immediately above. Let  $\psi$  be any nonnegative function on  $\Gamma$  which is bounded away from zero on  $F$ . Let further  $\theta$  be any complex-valued function on  $\Gamma$  such that

$$\theta(\gamma) = \psi(\gamma) |\hat{g}(\gamma)|^{1-2\eta(\gamma)} \cdot \text{sgn } \hat{g}(\gamma) \text{ for } \gamma \in F. \tag{A.22}$$

Then (A.21), (A.22) and Bochner's theorem combine to show that  $\theta$  is

not a Fourier-Stieltjes transform. Yet, if  $\psi$  is bounded, and if we define  $\theta(\gamma) = 0$  for  $\gamma \in \Gamma \setminus F$ , (A.20) and the fact that  $g \in C(G)$  ensure that

$$\theta \in \bigcap_{r>2} l^r(\Gamma). \tag{A.23}$$

We thus obtain explicit examples of functions  $\theta$  satisfying (A.23) which are not Fourier-Stieltjes transforms.

Note that, if every  $c_n$  is real and nonzero, an (unbounded)  $\psi$  can be chosen so as to make  $\text{Ran } \theta = \{-1, 1\}$ ; this yields explicit examples of  $\pm 1$ -valued functions  $\theta$  which are not Fourier-Stieltjes transforms. (These are, of course, also obtainable by starting with functions  $\text{sgn } \hat{h}$ , where  $h \in C(G)$ ,  $\hat{h}$  is real-valued and  $\hat{h} \notin l^1(\Gamma)$ .)

### BIBLIOGRAPHY

- [1] BRAINERD, B. and R. E. EDWARDS, Linear operators which commute with translations. Part 1: Representation theorems. *J. Australian Math. Soc.* 6 (1966), 289-327.
- [2] EDWARDS, R. E., *Functional Analysis: Theory and Applications*. Holt, Rinehart and Winston, New York (1965).
- [3] EDWARDS, R. E. and E. HEWITT, Pointwise limits for sequences of convolution operators. *Acta Math.* 113 (1965), 181-218.
- [4] FIGA-TALAMANCA, A. and G. I. GAUDRY, Multipliers and sets of uniqueness of  $L^p$ . *Michigan Math. J.* 17 (1970), 179-191.
- [5] GAUDRY, G. I., Bad behaviour and inclusion results for multipliers of type  $(p, q)$ . *Pacific J. Math.* 35 (1970), 83-94.
- [6] GEL'FAND, I. M. and G. E. SHILOV, *Generalized Functions, Volume 1*. Academic Press, New York (1964).
- [7] HEWITT, E. and K. A. ROSS, *Abstract Harmonic Analysis, Vols. I, II*. Springer-Verlag, Berlin (1963, 1970).
- [8] HEWITT, E. and H. S. ZUCKERMAN, On a theorem of P. J. Cohen and H. Davenport. *Proc. Amer. Math. Soc.* 14 (1963), 847-855.
- [9] PRICE, J. F., Some strict inclusions between spaces of  $L^p$ -multipliers. To appear in *Trans. Amer. Math. Soc.* 152 (1970).
- [10] RUDIN, W., *Fourier Analysis on Groups*. Interscience, New York (1962).
- [11] ZYGMUND, A., *Trigonometric Series, Vols. I & II*. Cambridge University Press (1959).
- [12] HOLLAND, Samuel S., Jr. A Hilbert space proof of the Banach-Steinhaus theorem. *Amer. Math. Monthly* 76 (1969), 40-41.
- [13] HALMOS, P. *A Hilbert space problem book*. Van Nostrand, Princeton, N.J. (1967).
- [14] HÖRMANDER, L., Estimates for translation invariant operators in  $L^p$  spaces. *Acta Math.* 104 (1960), 93-140.
- [15] BOURBAKI, N., *Espaces Vectoriels Topologiques, Ch. I, II*. Hermann et Cie, Paris (1953).
- [16] BROOKS, J. K. and J. MIKUSIŃSKI., On some theorems in functional analysis. *Bull. Acad. Pol. Sci.* (3) XVIII (1970), 151-155.
- [17] DUNKL, C. F. and D. E. RAMIREZ,  $L^p$  Multipliers on Compact Groups. To appear.

Department of Mathematics,  
Institute of Advanced Studies,  
Australian National University.

(Reçu le 1 septembre 1970)