

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 16 (1970)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: NAIVELY CONSTRUCTIVE APPROACH TO BOUNDEDNESS PRINCIPLES, WITH APPLICATIONS TO HARMONIC ANALYSIS
Autor: Edwards, R. E. / Price, J. F.
Kapitel: § 9. Discussion of case (ii) : G 0-dimensional
DOI: <https://doi.org/10.5169/seals-43866>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 28.03.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

In particular, taking $\Delta_j = \{n \in \mathbb{Z} : 2^j \leq n < 2^{j+1}\}$ it can be arranged that

$$\sum_{n \in \mathbb{Z}} \frac{\pm \hat{f}(n)}{(\log(2 + |n|))^\alpha}$$

diverges for any preassigned distribution of signs \pm and any preassigned $\alpha < \frac{1}{4}$.

Of course, much stronger results are derivable by using random (and unassignable!) changes of sign, but there seems little hope of making this even remotely constructive.

§ 9. Discussion of case (ii) : G 0-dimensional

9.1 In this case there is ([7], (7.7)) a base of neighbourhoods of zero in G formed of compact open subgroups W . For each such W the annihilator $\Delta = W^\circ$ in Γ of W is a finite subgroup of Γ . Define

$$k_W = \lambda_G(W)^{-1} \times \text{characteristic function of } W. \tag{9.1}$$

Then k_W is continuous, $k_W \geq 0$, $\int_G k_W d\lambda_G = 1$. The transform \hat{k}_W of k_W is plainly equal to unity on Δ . On the other hand, since W is a subgroup, we have for $a \in W$ and $\gamma \in \Gamma$

$$\begin{aligned} \hat{k}_W(\gamma) &= \int_G k_W(x) \overline{\gamma(x)} d\lambda_G(x) = \int_G k_W(x+a) \overline{\gamma(x)} d\lambda_G(x) \\ &= \int_G k_W(y) \overline{\gamma(y-a)} d\lambda_G(y) \\ &= \gamma(a) \hat{k}_W(\gamma), \end{aligned}$$

which shows that $\hat{k}_W(\gamma) = 0$ if $\gamma \in \Gamma \setminus \Delta$. Thus \hat{k}_W is the characteristic function of Δ , and so

$$k_W = D_{W^\circ}. \tag{9.2}$$

By (9.1) and (9.2), a routine argument shows that, if $1 \leq p < \infty$ and $f \in L^p(G)$, then

$$f = \lim_W S_{W^\circ} f \tag{9.3}$$

in $L^p(G)$; and that (9.3) holds uniformly for any continuous f .

9.2 PROOF OF 7.4 (ii). If Γ_0 is any countably infinite subgroup of Γ we can choose a sequence W_j of compact open subgroups of G such that

$W_{j+1} \subseteq W_j$ and $\Gamma_0 \subseteq \bigcup_{j=1}^{\infty} W_j^\circ$, where W_j° is a finite subgroup of Γ and $W_j^\circ \subseteq W_{j+1}^\circ$. The $\Delta_j = W_j^\circ \cap \Gamma_0$ satisfy (7.2) and, from (9.3),

$$f = \lim_j S_{\Delta_j} f \tag{9.4}$$

uniformly for any continuous f with $\text{sp}(f) \subseteq \Gamma_0$. This verifies the statements made in 7.4 (ii).

9.3 By using the results in [3], more can be said in case (ii) of 7.4; cf. [3], Theorem (2.9) and Example (4.8).

Let $f \in L^1(G)$ and let Γ_0 be any countable subgroup of Γ containing $\text{sp}(f)$. Choose the W_j as in 9.2. Then, apart from the fact that (W_j) is not in general a base at 0 in G (they can be chosen to be so if and only if G is first countable), (W_j) is an open-compact D'' -sequence ([3], p. 188). The proof of Theorem (2.5) of [3] is easily modified to show that

$$f(x) = \lim_{j \rightarrow \infty} S_{W_j^\circ} f(x) \tag{9.5}$$

holds for almost all $x \in G$. Moreover, Theorem (2.7) of [3] applies to show that the majorant function

$$S^* f(x) = \sup_{j \in \mathbb{N}} |S_{W_j^\circ} f(x)| \tag{9.6}$$

satisfies the estimates

$$\|S^* f\|_p \leq 2(p(p-1)^{-1})^{\frac{1}{p}} \|f\|_p \quad (1 < p < \infty) \tag{9.7}$$

$$\|S^* f\|_1 \leq 2 + 2 \int_G |f| \log^+ |f| d\lambda_G, \tag{9.8}$$

$$\|S^* f\|_p \leq 2(1-p)^{\frac{1}{p}} \|f\|_1 \quad (0 < p < 1). \tag{9.9}$$

In particular, the convergence in (9.5) is dominated whenever

$$|f| \log^+ |f| \in L^1(G).$$

A more immediate consequence of (9.1) and (9.2) is a strong version of localisability of the convergence of Fourier series: if $f \in L^1(G)$ vanishes a.e. on some neighbourhood of $x_0 \in G$, we can choose the W_j so that $S_{\Delta_j} f(x_0) = 0$ for every sufficiently large j . [A suitable choice of W_j may be made once for all, independent of f , if G is first countable.] Nothing similar is true for general G ; see, for example, [11], Vol. II, pp. 304-305.