A NAIVELY CONSTRUCTIVE APPROACH TO BOUNDEDNESS PRINCIPLES, WITH APPLICATIONS TO HARMONIC ANALYSIS

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GENERAL INTRODUCTION

This paper is partly pedagogical and expository. Thus Part 1 (§§ 1-4) presents a naively constructive approach to boundedness principles. Although this construction leads to results differing but slightly from the standard versions, we feel that this approach (which can be followed with no overt reference to category, barrelled spaces, and so on) offers some pedagogical and expository advantages. We emphasise that the level of constructivity is naive and not fundamental.

The remainder of the paper consists of applications of the constructive procedure. In Part 2 (§§ 5, 6) the applications yield improvements of recent results due to Price and to Gaudry concerning multipliers. In Part 3 (§§ 7-10) the applications are to convergence and divergence of Fourier series of continuous functions on compact Abelian groups. These results (which may be known to the afficionados but which, as far as we know, have not been published hitherto) characterise those compact Abelian groups having the property that every continuous function has a convergent Fourier series; and, in the remaining cases, applies the general method of Part 1 to construct continuous functions with divergent Fourier series.

PART 1: BOUNDEDNESS PRINCIPLES

§ 1. *Introduction and preliminaries*

Let E denote a locally convex space and P a set of bounded gauges on E; that is, each $f \in P$ is a function with domain E and range a subset of $[0, \infty)$ such that

$$f(x+y) \leqq f(x) + f(y) \quad (x, y \in E),$$

$$f(\alpha x) = \alpha f(x) \quad (x \in E, \alpha > 0),$$