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$$= \frac{(\alpha+1)}{(\beta+1)} \left\{ \int_0^1 t^z \, d\chi(t) + (\beta-\alpha) \int_0^1 v^{-\alpha-1} \, d\chi(v) \int_0^v t^{z+\alpha} \, dt \right\} =$$

$$= \frac{(\alpha+1)}{(\beta+1)} T(\alpha+1, \lambda, \beta+1, \mu; z) \left\{ 1 + \frac{\beta - \alpha}{z + \alpha + 1} \right\}, \qquad (18)$$

by the result obtained by replacing α, β by $\alpha+1$, $\beta+1$ in (14). It now follows at once from the definition of $T(\alpha, \lambda, \beta, \mu; z)$ that

$$\int_0^1 t^z \, d\psi(t) = T(\alpha, \lambda, \beta, \mu; z). \qquad (19)$$

We may suppose $\psi(t)$ normalised by taking

$$\psi(0) = 0; \quad \psi(t) = \tfrac{1}{2}(\psi(t+) + \psi(t-)) \quad (0 < t < 1).$$

If $\chi(\alpha, \lambda, \beta, \mu; t)$ is similarly normalised, it follows from (14) and (19) with the aid of the uniqueness theorem for Mellin transforms that

$$\psi(t) = \chi(\alpha, \lambda, \beta, \mu; t).$$

The proof of the theorem is thus completed.

5. It is easily seen that, whenever the transformation (7) is regular, it is also absolutely regular; that is, it transforms any absolutely convergent function (that is to say, a function of bounded variation in $(0, \infty)$) into an absolutely convergent function. The proof of the theorem therefore shows that the result remains true if we replace summability by absolute summability throughout.

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