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SIMPLE PROOFS OF TWO THEOREMS ON MINIMAL SURFACES

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To the memory of J. Karamata

1. INTRODUCTION

We will give simple proofs of the following uniqueness theorems on minimal surfaces:

THEOREM 1 (Bernstein). *Let* z = f (x, y) *be a minimal surface in euclidean three-space defined for all* x, y. *Then* f (x, y) *is a linear function.*

THEOREM 2. *A closed minimal surface of genus zero on the three-sphere must be totally geodesic and is hence a great sphere.*

Theorem 2 has been proved by Almgren [1] and Calabi [2].

2. PROOF OF THEOREM 1

Let

$$W = \left(1 + f_x^2 + f_y^2\right)^{\frac{1}{2}} \geqq 1 . \tag{1}$$

The proof is based on the identity

$$\Delta \log\left(1 + \frac{1}{W}\right) = K , \tag{2}$$

where Δ is the Laplacian relative to the induced riemannian metric of the minimal surface M and K is its Gaussian curvature.

Suppose (2) be true. Let ds be the element of arc on M. Introduce the conformal metric

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