Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 15 (1969)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE ASYMPTOTIC BEHAVIOR OF THE nth ORDER DIFFERENCE
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Bibliographie
DOI: https://doi.org/10.5169/seals-43202

For that purpose we have only to show

(20′) For any d positive there exist ε_d' and X_d such that $\bigcap_s N_{\tau,s}(x, \varepsilon_d', c)$ is non-empty for $\tau \in Q_d$ and $x > X_d$,

since in the same way in which we have deduced (18) from (19) and (20) we can deduce (18′) from (19) and (20′), taking $\varepsilon_d = 2^n \varepsilon_d'$.

Investigating the proof of (20), we see that in order to prove (20′) we need only

(32) For every d positive there exist ε_d' and Y_d such that

$$m(N(x, \varepsilon_d', c)) > m(Q_c) - d^n \quad \text{for} \quad x > Y_d .$$

Let us assume that (32) does not hold. Then there exists a sequence $\{x_k\}$, $x_k \to \infty$ such that

$$m(N(x_k, k, c)) \leq m(Q_c) - d^n$$

for $k = 1, 2, \ldots, n$. It follows that the set $N = \bigcup_{i=1}^{\infty} \bigcap_{k=i}^{\infty} N_k$, where $N_k = N(x_k, k, c)$, is a proper subset of Q_c. So there exists $\tau_0 \in Q_c$ such that $\tau_0 \notin N_k$ for infinitely many k. This means that $|\Delta_{\tau_0}^{(n)} f(x_k)| \geqslant k$ for infinitely many k, which is in contradiction with (31).

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(Reçu le 15 janvier 1969)

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