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The set $G^* \subset G$ is open and $R^{**} = \{V_1, ..., V_{\iota^*}\}$ an open covering of G^* such that $V_{\iota} \subset \subset U_{\iota}$ for $\iota \in \{1, ..., \iota^*\}$. We have:

Cartan's Theorem. There exists a constant K such that if $\xi \in Z^l(R^*, q0)$ then $\xi \mid R^{**} = \delta \eta$ where $\eta \in C^{l-1}(R^{**}, q0)$ and $||\eta|| \leq K ||\xi||$ for $l \geqslant 1$.

This is a simple consequence of Theorem B and Banach's open mapping theorem.

Now we apply Cartan's theorem. We keep the notations as above. Let $\hat{G} = G \times E^n(\rho)$ and put $\hat{R}^* = \{U_\iota \times E^n(\rho)\}$. Now \hat{R}^* is a Stein covering of \hat{G} . Let $\hat{G}^* = G^* \times E^n(\rho)$ and $\hat{R}^{**} = \{V_\iota \times E^n(\rho)\}$. Let $\hat{\xi} \in Z^l(\hat{R}^*, q\emptyset)$ and write $\hat{\xi} = \sum \xi_{(v)} (t/\rho)^v$ with $\xi_{(v)} \in Z^l(R^*, q\emptyset)$. We assume $\|\hat{\xi}\|_{\rho} = \sup_v \|\xi_{(v)}\| < \infty$. Now Cartan's theorem gives $\xi_{(v)} \|R^{**} = \delta \eta_v \text{ with } \eta_v \in C^{l-1}(R^{**}, q\emptyset) \text{ and } \|\eta_v\| \leqslant K \|\xi_{(v)}\| < \infty$. It follows that $\hat{\eta} = \sum_v \eta_v (t/\rho)^v$ is well defined in $C^{l-1}(\hat{R}^{**}, q\emptyset)$ and by definition we have $\|\hat{\eta}\|_{\rho} \leqslant K \|\hat{\xi}\|_{\rho}$.

SMOOTHING

We are given a sequence of admissible refinements of measure coverings in $X(\rho_1)$. Here $\rho_1 < \rho_0 = \min \rho_1$ as usual. Let l be a fixed integer $\geqslant 1$. We are given $\mathfrak{B}^* \ll \mathfrak{B}' = \mathfrak{B}_{3l} \ll \mathfrak{B}_{3l-1} \ll \ldots \ll \mathfrak{B}_1 \ll \mathfrak{B}_0 \ll \mathfrak{B} \ll \mathfrak{U}^* \ll \mathfrak{U} = \mathfrak{U}_{3l} \ll \ldots \ll \mathfrak{U}_0 \ll \mathfrak{U}'$. Here it is also required that $(\mathfrak{B}_{v+1},\mathfrak{U}_{v+1}) \ll (\mathfrak{B}_v,\mathfrak{U}_v)$; $(\mathfrak{B}^*,\mathfrak{U}^*) \ll \mathfrak{U}'$. These extra conditions mean: 1) $\hat{U}^{(v+1)}_{i_0} \ldots_{i_K} \cap \hat{V}^{(v+1)}_{i_0} \ldots_{i_k} \cap \hat{V}^{(v)}_{i_0} \ldots_{i_l}$ for each $i \in \{i_0, \ldots, i_K\}$ and 2) $(U^{(v+1)}_{i_0} \ldots_{i_K} \cap V^{(v+1)}_{i_0} \ldots_{i_K} \cap V^{(v)}_{i_0} \ldots_{i_l})_i$ for all $i,j \in \{i_0,\ldots,i_K,\iota_0,\ldots\iota_l\}$. Recall that all operations are done with respect to ρ_1 . Let us put $\hat{R}^{(v)}_{i_0} \ldots_{i_K,\iota_0,\ldots\iota_K} = \hat{U}^{(v)}_{i_0} \ldots_{i_K} \cap \hat{V}^{(v)}_{i_0} \ldots_{i_K} \cap \hat{V}^{(v$

Norm in $C_{\nu}^{k,\kappa}$: Let $\xi \in C_{\nu}^{k,\kappa}$; we put

$$\begin{split} & || \stackrel{\hat{\varsigma}}{\xi} ||_{\rho} = \max_{i,(i_0,\ldots i_k,\iota_0,\ldots,\iota_{\kappa})} \big\{ \, || \stackrel{\hat{\varsigma}}{\xi}_{i_0\ldots i_k\,\iota_0\ldots\iota_{\kappa}} \big| \big(R_{i_0\ldots i_k\,\iota_0\ldots\iota_{\kappa}}^{(\nu+1)} \big)_i(\rho) \, ||_i \text{ with } i \in \{i_0,\ldots,i_k\} \big\}. \text{ Here} & p_1 \text{ and } R_{i_0\ldots i_k}^{(\nu+1)},_{\iota_0\ldots\iota_{\kappa}} = U_{i_0\ldots i_k}^{(\nu+1)} \cap V_{\iota_0\ldots\iota_{\kappa}}^{(\nu+1)} \text{ and } || \, ||_i \text{ is taken with respect to the chart } \mathscr{W}_i \text{ as usual.} \end{split}$$

SMOOTHING LEMMA: Let $\kappa > 0$. There exists a constant K such that: If $\hat{\xi} \in C_{\nu}^{k,\kappa}$ with $\hat{\partial}\hat{\xi} = 0$ and $\|\hat{\xi}\|_{\rho} < \infty$ then we can find $\hat{\eta} \in C_{\nu+3}^{k,\kappa-1}$ such that $\hat{\xi} \|C_{\nu+3}^{k,\kappa} = \hat{\partial}\hat{\eta}$ and $\|\hat{\eta}\|_{\rho} \leqslant K \|\hat{\xi}\|_{\rho}$. Here $\rho \leqslant \rho_2 = \gamma \rho_1$ with $0 < \gamma < 1$ and K depends only on ρ_2 .

Proof. Let us fix $i_0, ..., i_k$ in the following discussion. Let $G = U^{(v+1)}_{i_0...i_k}$ and put $\hat{G} = (G)_i (\rho_1)$ for some $i \in \{i_0, ..., i_k\}$ which is also fixed now. Now G is Stein in X_0 and \hat{G} is Stein in X. We put $R^* = G \cap \mathfrak{B}_{v+1}$ which is a Stein covering of G. Also $\hat{R}^* = \{(G \cap V_{\iota}^{(v+1)})_i (\rho_1)\}_{\iota=1,...,\iota^*}$ is a Stein covering of \hat{G} . Let $\hat{\xi} = \{\hat{\xi}_{i_0,...i_k,\iota_0...\iota_k}\}$. Now we look at the elements of $\{\hat{\xi}_{i_0,...i_k,\iota_0...\iota_k}\} = \hat{\xi}_{i_0,...i_k} \in Z^{\kappa}(\hat{R}^*, \mathbf{F})$. Here $i_0,...i_k$ is fixed as above. We get a cocycle because we have assumed that $\hat{\partial}\hat{\xi} = 0$. More precisely we have considered the restriction of $\hat{\xi}_{i_0,...i_k,\iota_0,...\iota_k}$ to \hat{R}^* . We must verify that this restriction is possible.

Verification: By definition of $Z^{\kappa}(\hat{R}^*, \mathbf{F})$ we have to look at sets of the following type: (these are the sets where the cross-sections are defined) $(G \cap V^{(v+1)})_i \cap \ldots \cap (G \cap V^{(v+1)})_i = (G \cap V^{(v+1)})_i = (R^{(v+1)}_{i_0 \dots i_{\kappa}})_i = (R^{(v+1)}_{i_0 \dots i_{k} i_0 \dots i_{\kappa}})_i$. Now by 2) we have $(R^{(v+1)}_{i_0 \dots i_k i_0 \dots i_{\kappa}})_i \subseteq \bigcap_j (R^{(v)}_{i_0} \dots i_{k} i_0 \dots i_{\kappa})_j \subseteq (U^{(v)}_{i_0})_{i_0} \cap \ldots \cap (V^{(v)}_{i_{\kappa}})_{i_{\kappa}} = \hat{R}^{(v)}_{i_0} \dots i_{k} i_0 \dots i_{\kappa}$. Q.E.D.

Now we put $G^* = U^{(\nu+2)}_{i_0...i_k} \subset G$. We let $\hat{R}^{**} = \{(G^* \cap V_{\iota}^{(\nu+2)})_i\}_{\iota=1,...,\iota^*}$. The system \hat{R}^{**} is a Stein covering of $(G^*)_i$. We are in a good position now. For we are given $\hat{\xi}_{i_0,...i_k} \in Z^{\kappa}(\hat{R}^*, \mathbf{F})$. Here \hat{R}^* is a Stein covering of \hat{G} and \hat{G} is a Stein manifold. We are working in the chart \mathcal{W}_i where the usual identifications are used. Hence we arrive at the following situation: G is a Stein manifold with a Stein covering $R^* = \mathfrak{V}_{\nu+1} \cap G$. Also $G^* \subset G$ and $R^{**} = \mathfrak{V}_{\nu+2} \cap G^*$ is a Stein covering of G^* such that $R^{**} \subset G^*$. The cocycle $\hat{\xi}_{i_0,...i_k}$ is now considered as an element of $Z^{\kappa}(\hat{R}^*, q0)$ which

we simply call $\hat{\xi}_{i_0...i_k}$ again. Now we apply the result after Cartan's theorem. Hence we can find a constant K such that for every $\rho \leqslant \rho_2$ we get $\eta \in C^{\kappa-1}(\hat{R}^{**},q\mathcal{O})$ and $\|\eta\|_{\rho} \leqslant K\|\hat{\xi}_{i_0...i_k}\|_{\rho}$ with $\partial \eta = \hat{\xi}_{i_0...i_k}$. But this means precisely that we can find $\hat{\eta}_{i_0...i_k} \in C^{\kappa-1}(\hat{R}^{**}(\rho),\mathbf{F})$ such that $\|\hat{\eta}_{i_0...i_k}\|_{i,\rho} \leqslant K\|\hat{\xi}_{i_0...i_k}\|_{i,\rho}$ with $\hat{\xi}_{i_0...i_k} = \partial \hat{\eta}_{i_0...i_k}$. We have only constructed $\hat{\eta}_{i_0...i_k}$ using a fixed $i \in \{i_0,...,i_k\}$. Now we must let $(i_0,...,i_k)$ vary. For each $(i_0,...,i_k)$ we choose some i which only depends on the unordered (k+1)-tupel $(i_0,...,i_k)$ and construct an element $\hat{\eta}_{i_0...i_k}$ as above. Now we can restrict everything to $C^{k,\kappa-1}_{\nu+3}$.

Verification: Consider a set where cross-sections over $C^{k,\kappa-1}_{\nu+3}$ have to be defined, i.e. a set $\hat{U}^{(\nu+3)}_{i_0\dots i_k}\cap\hat{V}^{(\nu+3)}_{i_0\dots i_\kappa}$. But by 1) follows $\hat{U}^{(\nu+3)}_{i_0\dots i_k}\cap\hat{V}^{(\nu+3)}_{i_0\dots i_\kappa}\subset (R^{(\nu+2)}_{i_0\dots i_k},_{i_0\dots i_k})_i$ for each $i\in\{i_0,\dots,i_\kappa\}$. This inclusion shows that we get a well defined element $\hat{\eta}\in C^{k,\kappa-1}_{\nu+3}$ by restricting the elements $\hat{\eta}_{i_0,\dots i_k}$ to $C^{k,\kappa-1}_{\nu+3}$. We find that $\hat{\xi}\mid C^{k,\kappa}_{\nu+3}=\hat{\partial\eta}$ now. The norm inequalities are not obvious, but recalling how $\hat{\eta}$ is constructed here it is seen that we can apply Theorem I to obtain the required estimate.

SMOOTHING THEOREM. There exists a constant K such that: If $\hat{\xi} \in Z^l(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ with $\|\hat{\xi}\|_{\rho} < \infty$ then we can find $\hat{\xi}^* \in Z^l(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ and $\hat{\eta} \in C^{l-1}(\hat{\mathfrak{B}}'(\rho), \mathbf{F})$ for which $\hat{\xi}^* |\hat{\mathfrak{B}}'(\rho) = \hat{\xi} |\hat{\mathfrak{B}}'(\rho) + \hat{\delta \eta}$ and $\|\hat{\xi}^*\|_{\rho}$ and $\|\hat{\eta}\|_{\rho} \leqslant K \|\hat{\xi}\|_{\rho}$. Here $\rho \leqslant \rho_2 < \rho_1$ and K only depends on ρ_2 .

Proof. Before we can use the double complex $\{C_{\nu}^{k,\kappa}\}$ we must introduce two " ε -maps". To define the ε_1 -map, let $Z_{\nu}^{k,\kappa} \subset C_{\nu}^{k,\kappa}$ consist of all $\hat{\xi} \in C_{\nu}^{k,\kappa}$ such that $\delta \hat{\xi} = \hat{\partial} \hat{\xi} = 0$. Now we shall define the ε_1 -map: ε_1 : $Z^l(\mathfrak{D}, \mathbf{F}) \to Z_0^{0,l}$. A section belonging to an element of $C_0^{0,l}$ is defined on some set $U_{i_0}^{(0)} \cap \hat{V}_{i_0}^{(0)} \dots_{i_l} \subset \hat{V}_{i_0 \dots i_l}$ where sections of elements of $Z^l(\mathfrak{D}, \mathbf{F})$ are defined. Hence we get a natural restriction map ε_1 which also maps cocycles into cocycles. It is easy to verify that $\|\varepsilon_1(\hat{\xi})\|_{\rho} \leqslant K\|\hat{\xi}\|_{\rho}$. Theorem I can be used because $(U_i^{(1)} \cap V_{i_0}^{(1)} \dots_{i_l})_i \subset (V_{i_0 \dots i_l}^{(0)} \dots_{i_l})_i$ for every i and every $i \in \{\iota_0, \dots \iota_l\}$. Recall that the norm in $Z^l(\mathfrak{D}, \mathbf{F})$ is defined with respect to

 $\widehat{\mathfrak{B}}_0 \text{ here. The "} \ \varepsilon_2\text{-map ": we shall construct a map } \ \varepsilon_2\colon Z_{3l}^{l,0}\to Z^l(\widehat{\mathfrak{U}},\mathbf{F}).$ Let $\widehat{\boldsymbol{\xi}}=\{\widehat{\xi}_{i_0,\ldots i_l,\iota_0}\}\in Z_{3l}^{l,0}.$ Here $\widehat{\boldsymbol{\xi}}_{i_0,\ldots i_l,\iota_0}$ is defined on $\widehat{R}_{i_0\ldots i_l,\iota_0}^{(3l)}.$ Because $\widehat{\partial}\widehat{\boldsymbol{\xi}}=0$ we see that the elements $\widehat{\boldsymbol{\xi}}_{i_0,\ldots i_l,\iota_0}$ are independent of ι_0 . Now $\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}{\overset{\iota^*}}{\overset{\iota^*}}{\overset{\iota^*}}{\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}{\overset{\iota^*}}}}\overset{\iota^*}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}{\overset{\iota^*}}}\overset{\iota^*}}{\overset{\iota^*}}}$

Verification. A computation of $\|\varepsilon_2(\hat{\xi})\|_{\rho}$ involves the following: $\varepsilon_2(\hat{\xi}) = \{\xi_{i_0}^{(2)}..._{i_l}\}$. Look at some $\xi_{i_0...i_l}^{(2)}$ in the chart \mathcal{W}_i with $i \in \{i_0, ..., i_l\}$. We write $\hat{\xi}_{i_0...i_l}^{(2)} = \sum a_v (t/\rho)^v$ over $(U_{i_0}^*..._{i_l})_i$ and compute $\sup_v \|a_v (U_{i_0}^*..._{i_l})\|$. A computation of $\|\hat{\xi}\|_{\rho}$ involves the following: Look at $\hat{\xi}_{i_0...i_l}$ over $(U_{i_0}^*..._{i_l}\cap V_{i_l}^*)\|$ in a chart W_i . Here ι is fixed. We write $\hat{\xi}_{i_0...i_l,\iota} = \sum a_v^{(\iota)} (t/\rho)^v$ and compute $\sup_v \|a_v^{(\iota)} (U_{i_0}^*..._{i_l}\cap V_{\iota}^*)\|$. Now $\bigcup_v V_{\iota}^*$ covers X_0 . Hence we would have $\sup_v \|a_v^{(\iota)} (U_{i_0}^*..._{i_l}\cap V_{\iota}^*)\| = \sup_v \|a_v (U_{i_0}^*..._{i_l})\|$ if $a_v = a_v^{(\iota)}$ in $U_{i_0}^*..._{i_l}\cap V_{\iota}^*$ hence we have $\|\varepsilon_2(\hat{\xi})\|_{\rho} \leqslant \|\hat{\xi}\|_{\rho}$.

Now we are ready to start the proof of the smoothing theorem. We let K denote a constant, which may be different at different occurences. We also introduce a double complex $\{\tilde{C}^{k,\kappa}_{\ \nu}\}$ using $(\mathfrak{B},\mathfrak{B})$, i.e. it is defined just as the previous double complex was, using \mathfrak{B} -sets instead of \mathfrak{U} -sets. We shall inductively construct the following elements:

$$\begin{split} \hat{\xi}_{v} &= \{\hat{\xi}_{i_{0}...i_{v}}, {}_{\iota_{0}}...{}_{\iota_{l-v}}\} \in Z^{v,l-v}_{3v} \\ \tilde{\xi}_{v} &= \{\tilde{\xi}_{i_{0}...i_{v}}, {}_{\iota_{0}...\iota_{l-v}}\} \in \tilde{Z}^{v,l-v}; \, v = 0,...,l \\ \hat{\eta}_{v} &= \{\hat{\eta}_{i_{0}...i_{v-1}}, {}_{\iota_{0}...\iota_{l-v}}\} \in C^{v-1}_{3v}, {}_{l-v} \\ \tilde{\eta}_{v} &= \{\tilde{\eta}_{i_{0}...i_{v-1}}, {}_{\iota_{0}...\iota_{l-v}}\} \in \tilde{C}^{v-1}_{3v}, {}_{v-1}; \quad v = 1,...,l \end{split}$$

The construction: $\hat{\xi} \in Z^{l}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ is given. The whole construction is done using ρ instead of ρ_1 and we omit ρ to simplify the notation. We put $\varepsilon_1(\hat{\xi}) = \hat{\xi}_0 \in Z_0^{0,l}$. Now we apply the Smoothing Lemma and get η_1 such that $\widehat{\partial \eta_1} = \widehat{\xi}_0$ with $\|\widehat{\eta_1}\|_{\rho} \leqslant K \|\widehat{\xi}_0\|_{\rho} \leqslant K \|\widehat{\xi}\|_{\rho}$. Put $\widehat{\xi}_1 = \widehat{\delta \eta_1}$. Obviously $\|\hat{\xi}_1\|_{\rho} \leqslant K \|\hat{\eta}_1\|_{\rho}$. Inductively we find $\delta \hat{\eta}_{\nu} = \hat{\xi}_{\nu-1}$ and we put $\xi_{\nu} = \delta \eta_{\nu}$ where η_{ν} are found from the Smoothing Lemma. Finally we get $\hat{\xi}_l$ and we have $\|\hat{\xi}_l\|_{\rho} \leqslant K \|\hat{\xi}\|_{\rho}$. Now we define $\tilde{\xi}_{\nu}$ and $\tilde{\eta}_{\nu}$ as follows. Put $\tilde{\xi}_0 = \hat{\xi}_0$ where $\tilde{\xi}_0 \in Z_0^{0,l}$ is obtained by natural restriction of $\hat{\xi}_0$. Put $\tilde{\eta}_v = (-1)^v \{\hat{\xi}_{i_0...i_{v-1}}, \hat{\xi}_{i_0...i_{v-1}}\}$ which is well defined with respect to $(\mathfrak{B}_{3\nu}, \mathfrak{B}_{3\nu})$ by taking natural restrictions. Put $\tilde{\xi}_{\nu} = \delta \tilde{\eta}_{\nu}$ for $\nu = 1, ..., l$. A computation shows that $\xi_{v-1} = \partial \eta_v$ when v = 1, ..., l. Notice that this is trivial when v = 1. In the following discussion each η_v is restricted to $(\mathfrak{B}_{3\nu}, \mathfrak{B}_{3\nu})$. We have $\partial (\tilde{\eta}_1 - \hat{\eta}_1) = 0$. Hence we find $\tilde{\eta}_1 - \tilde{\eta}_1 = \partial \tilde{\gamma}_1$ by the Smoothing Lemma. Now we define $\tilde{\gamma}_{\nu}$ such that $\partial \tilde{\gamma}_{\nu} = \tilde{\eta}_{\nu} - \tilde{\eta}_{\nu} - \delta \tilde{\gamma}_{\nu-1}$ inductively. This is possible because $\partial (\eta_{\nu} - \eta_{\nu} - \delta \tilde{\gamma}_{\nu-1}) = 0$, for we have $\partial(\overset{\sim}{\eta_{\nu}} - \overset{\sim}{\eta_{\nu}} - \overset{\sim}{\delta \overset{\sim}{\eta_{\nu-1}}}) = \overset{\sim}{\xi_{\nu-1}} - \overset{\sim}{\xi_{\nu-1}} - \delta \overset{\sim}{\partial \overset{\sim}{\eta_{\nu-1}}} = \delta \overset{\sim}{\eta_{\nu-1}} - \delta \overset{\sim$ $-\delta(\tilde{\eta}_{v-1}-\hat{\eta}_{v-1})=0. \text{ We get finally } \tilde{\gamma}_{l-1}\in \tilde{C}^{l-2,0} \text{ and then } \delta\tilde{\gamma}_{l-1}\in$ $\in \tilde{C}^{l-1,0}$. We have $\partial (\overset{\sim}{\eta}_l - \overset{\sim}{\eta}_l - \overset{\sim}{\delta \gamma}_{l-1}) = 0$. Therefore we can put $\overset{\sim}{\gamma}_l =$ $= \varepsilon_2 (\widetilde{\eta}_l - \widetilde{\eta}_l - \widetilde{\delta \gamma}_{l-1}). \text{ It follows that } \widetilde{\gamma}_l \in C^{l-1} (\mathfrak{B}_{3l}) \text{ and } \widetilde{\delta \gamma}_l = \varepsilon_2 (\widetilde{\xi}_l - \widehat{\xi}_l).$ We have $\varepsilon_2(\tilde{\xi}_l) = -\hat{\xi} \mid \mathfrak{B}'$ and for $\varepsilon_2(\hat{\xi}_l) = -\hat{\xi}^*$ and $\hat{\eta} = \tilde{\gamma}_l$ the required equation $\hat{\xi}^* = \hat{\xi} + \hat{\delta \eta}$. The estimates follow immediately from the construction and the Smoothing Lemma.