$= \{ \hat{U}_\iota (\rho) \}_1^{\iota^*}$ is a Stein covering of $X(\rho)$. We say then that $\hat{\mathfrak{U}} (\rho)$ is a measure covering of $X(\rho)$.

*Admissible refinements of measure coverings.* Let $\hat{\mathfrak{U}} (\rho)$ and $\hat{\mathfrak{U}}^* (\rho)$ be two measure coverings of $X(\rho)$. We say that $\hat{\mathfrak{U}}^* (\rho)$ is an admissible refinement of $\hat{\mathfrak{U}} (\rho)$ if the following conditions hold:

1) $U_\iota^* \subset \subset U_\iota$ for each $\iota$.

2) If $U_{\iota_0 \ldots \iota_\lambda}^* = U_{\iota_0}^* \cap \ldots \cap U_{\iota_\lambda}^*$ we put $(U_{\iota_0 \ldots \iota_\lambda}^*)_\nu = \Phi_\nu^{-1} \left( U_{\iota_0 \ldots \iota_\lambda}^* \times E^n (\rho) \right)$ for each $\nu \in \{ \iota_0 \ldots \iota_\lambda \}$. It is now required that $(U_{\iota_0 \ldots \iota_\lambda}^*)_\nu \subset (U_{\iota_0 \ldots \iota_\lambda})_\mu$ for all $\nu, \mu \in \{ \iota_0 \ldots \iota_\lambda \}$.

3) $\hat{U}_{\iota_0 \ldots \iota_\lambda}^* = \hat{U}_{\iota_0}^* \cap \ldots \cap \hat{U}_{\iota_\lambda}^* \subset (U_{\iota_0 \ldots \iota_\lambda})_\mu$ for each $\mu \in \{ \iota_0 \ldots \iota_\lambda \}$.

### EXISTENCE OF ADMISSIBLE REFINEMENTS OF MEASURE COVERINGS

*Existence Theorem.* For every fixed integer $s$ we can find, for some $\rho > 0$, a sequence $\mathfrak{U}_s \ll \mathfrak{U}_{s-1} \ll \ldots \ll \mathfrak{U}_1 \ll \mathfrak{U}_0$ of finer measure coverings of $X(\rho)$ each of which is an admissible refinement of the following.

*Proof.* We first construct a measure covering of $X(\rho)$ for some $\rho < \min \rho_\iota$. Let $\mathfrak{U}_0 = \{ \mathfrak{U}_\iota \}_1^{\iota^*}$ be a Stein covering of $X_0$ such that $U_\iota \subset \subset W_\iota$ for $\iota \in \{ 1, \ldots, \iota^* \}$. Choose a fixed $\rho_0 < \min \rho_\iota$. Now the open sets $\Phi_\iota^{-1} \left( U_\iota \times E^n (\rho_0) \right)$ cover $X_0$ and hence they also cover $X(\rho)$ for some sufficiently small $\rho$. Hence $\mathfrak{U}_0$ defines a measure covering of $X(\rho)$. It is also clear that $\mathfrak{U}_0$ defines a measure covering of $X(\rho')$ for each $\rho' \leqslant \rho$. Let us now construct $\mathfrak{U}_1$. We let $\mathfrak{U}^* = \{ U_\iota^* \}_1^{\iota^*}$ be a Stein covering such that $U_\iota^* \subset \subset U_\iota$ always holds. Now we can find $\rho_1 \leqslant \rho$ such that $\{ \hat{U}_\iota^* (\rho_1) = \Phi_\iota^{-1} \left( U_\iota^* \times E^n (\rho_1) \right) \}_1^{\iota^*}$ cover $X(\rho_1)$. Hence $\hat{\mathfrak{U}}^* (\rho_1)$ and $\hat{\mathfrak{U}} (\rho_1)$ are measure coverings of $X(\rho_1)$. But we do not yet know if $\hat{\mathfrak{U}}^* (\rho_1) \ll \hat{\mathfrak{U}} (\rho_1)$. We claim that if $\rho_2 \leqslant \rho_1$ is sufficiently small then $\hat{\mathfrak{U}}^* (\rho_2) \ll \hat{\mathfrak{U}} (\rho_2)$. For suppose this is false. Say that 2) fails for $\hat{\mathfrak{U}}^* (\rho_2)$ and $\hat{\mathfrak{U}} (\rho_2)$ when $0 < \rho_2 \leqslant \rho_1$. Hence $\Phi_\nu^{-1} \left( U_{\iota_0 \ldots \iota_\lambda}^* \times E^n (\rho_2) \right) - \Phi_\mu^{-1} \left( U_{\iota_0 \ldots \iota_\lambda} \times E^n (\rho_2) \right)$ are non empty for suitable indices while $\rho_2 \to 0$. Choose a point $x_t$ from each of these sets. Because $x_t \in X(\rho_1)$ which is relatively compact we may assume that $x_t \to x_0$. Obviously we get $x_0 \in \overline{U_{\iota_0 \ldots \iota_\lambda}^*} - U_{\iota_0 \ldots \iota_\lambda}$, a contradic-

tion because $\overline{U^*_{\iota_0 \cdots \iota_\lambda}} \subset \overline{U^*_{\iota_0}} \cap \ldots \cap \overline{U^*_{\iota_\lambda}} \subset U_{\iota_0 \cdots \iota_\lambda}$. In the same way we can prove that condition 3) is satisfied if $\rho_2$ is sufficiently small and the theorem is clear.

## GENERAL THEORY

Let $G$ be an analytic manifold. We put $\hat{G} = G \times E^n(\rho_1)$ where $\rho_1$ is an $n$-tuple of positive numbers. Let $\pi \colon \hat{G} \to E^n(\rho_1)$ and $\mathfrak{P} \colon \hat{G} \to G$ be the projection maps. $\hat{G}^* \subset \hat{G}$ denotes an open subset and $G^* = \hat{G}^* \cap G \times \{0\}$.

The set $G^*$ can be identified with an open subset of $G$. We denote by $\alpha \colon G^* \times E^n(\rho_1) \to \hat{G}^*$ a biholomorphic fiber preserving map, i.e. $\pi \circ \alpha = \pi^*$ where $\pi^* \colon G^* \times E^n(\rho_1) \to E^n(\rho_1)$ is the natural projection. Let $\rho \leqslant \rho_2 = \gamma \rho_1 < \rho_1$ where $0 < \gamma < 1$ is a fixed number. We put $\hat{G}(\rho) = G \times E^n(\rho)$. If $f$ is a holomorphic function on $\hat{G}(\rho)$ we write $f = \sum a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. We define the norm $\|f\|_\rho$ of $f$ by $\|f\|_\rho = \sup_\nu \{\sup |a_\nu(G)|\}$.

If $f \in I(\hat{G}(\rho))$ we see that $f \circ \alpha$ is a well defined function on $G^* \times E^n(\rho)$ because $\alpha$ is fiber preserving. We define $\|f \circ \alpha\|_\rho$ using $G^*$ instead of $G$ as above. We have the proposition:

*Proposition 1.* There exists a constant $K$ such that $\|f \circ \alpha\|_\rho \leqslant K \|f\|_\rho$ where $K = K(\rho_2)$ is independent of $\rho \leqslant \rho_2$.

*Proof.* We write $f = \sum_{|\nu| = 0}^{\infty} a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. Now we get $f \circ \alpha = \sum (a_\nu \circ \mathfrak{P} \circ \alpha)(t/\rho)^\nu$ because $\alpha$ is fiber preserving. Since $\mathfrak{P}(\hat{G}^*) \subset G$ we get $|a_\nu \circ \mathfrak{P}(\hat{G}^*)| \leqslant |a_\nu(G)| \leqslant \|f\|_\rho$. Now $a_\nu \circ \mathfrak{P} \circ \alpha$ admits a Taylor series: $a_\nu \circ \mathfrak{P} \circ \alpha = \sum C_{\nu\lambda}(t/\rho)^\lambda$ with $C_{\nu\lambda} \in I(G^*)$. Since $|\sum C_{\nu\lambda}(t/\rho)^\lambda| \leqslant \|f\|_\rho$ in $G^* \times E^n(\rho_1)$ and $\rho \leqslant \rho_2 = \gamma \rho_1$ Cauchy's inequalities give us $|C_{\nu\lambda}(G^*)| \leqslant \|f\|_\rho \gamma^{|\lambda|}$. Let us put $b_\mu = \sum_{\nu + \lambda = \mu} C_{\nu\lambda}$. We get $|b_\mu(G^*)| \leqslant \|f\|_\rho \sum \gamma^{|\lambda|} = \|f\|_\rho (1 - \gamma)^{-n} = K \|f\|_\rho$. Now we can write $f \circ \alpha = \sum_\nu a_\nu \circ \mathfrak{P} \circ \alpha (t/\rho)^\nu = \sum_{\lambda, \nu} C_{\nu\lambda}(t/\rho)^\lambda (t/\rho)^\nu = \sum_\mu b_\mu(t/\rho)^\mu$. By definition we have $\|f \circ \alpha\|_\rho = \sup_\mu |b_\mu(G^*)| \leqslant K \|f\|_\rho$.

Let us now consider $\mathbf{h} = (h_{\nu\mu})$ which is a $q \times q$ matrix with $h_{\nu\mu} \in I(\hat{G})$. The $h_{\nu\mu}$ are also assumed to be bounded on $\hat{G}$.