

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 14 (1968)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FLATNESS AND PRIVILEGE
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Kapitel: §1. Morphisms from an analytic space into $B(K)$
DOI: <https://doi.org/10.5169/seals-42343>

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whenever $x \in \overset{\circ}{K}$, we get $a = \inf_K |h(x)| > 0$. Hence $\|hf\| = \sup_K |hf(x)| \geq \geq a \sup_K |f(x)| = a \|f\|$.

(i') \Rightarrow (ii). Suppose that $X \cap \overset{\circ}{K} \neq \emptyset$ and $x = (x_1, x_2) \in X \cap \overset{\circ}{K}$. We choose an analytic function $f_1 : U_1 \rightarrow \mathbf{C}$, where $U_1 \supset K_1$, and U_1 is open, such that $f_1(x_1) = 1$, $|f_1(z)| < 1$ if $z \in K_1$, $z \neq x_1$. Similarly we choose an analytic function $f_2 : U_2 \rightarrow \mathbf{C}$, with the same properties. Consider the function $f \in B(K) : (z_1, z_2) \rightarrow f_1(z_1)f_2(z_2)$. Since $h(x) = 0$ it follows that the sequence $\{hf^n\}$ converges pointwise to 0 in K .

Applying Dini's theorem we get $\|hf^n\| \rightarrow 0$. From the inequality $a \|f^n\| \leq \|hf^n\|$ we get $\|f^n\| \rightarrow 0$, which is a contradiction, because for every $n : f^n(x) = 1$.

(b) Use the Weierstrass preparation theorem (extended form).

Question. Does the condition (ii) imply that $h : B(K) \rightarrow B(K)$ is a split monomorphism?

IV. FLATNESS AND PRIVILEGE

§ 1. Morphisms from an analytic space into $B(K)$

Let S be an analytic space and K a polycylinder in an open set $U \subset \mathbf{C}^n$. We want to construct an \mathcal{O}_S -algebra homomorphism $\phi : \mathcal{O}_{S \times U}(S \times U) \rightarrow \mathcal{H}(S; B(K))$.

(a) Consider first $S = U' \subset \mathbf{C}^m$, U' -open. If $h \in \mathcal{O}_{U' \times U}(U' \times U)$ and $s \in U'$, $x \in K$, define $(\phi(h)(s))(x) = h(s, x)$. Using the Cauchy integral, one can show that $\phi(h)$ is analytic. On the other hand it's obvious that ϕ is an $\mathcal{O}_{U'}$ -algebra homomorphism.

(b) Let S have a special model in the polydisc Δ in \mathbf{C}^m , defined by a sheaf \mathcal{I} of ideals of \mathcal{O}_Δ , and let \mathcal{J} be generated by f_1, \dots, f_p , V -a polycylinder neighbourhood of K in U . By Cartan's theorem B for a polycylinder,

the sequence $0 \rightarrow \mathcal{I}(\Delta \times V) \rightarrow \mathcal{O}(\Delta \times V) \xrightarrow{\pi} \mathcal{O}(S \times V) \rightarrow 0$ is exact. If we denote by $\tilde{\pi}$ the projection $\mathcal{H}(\Delta, B(K)) \rightarrow \mathcal{H}(S, B(K))$, $(f_1, \dots, f_p) \cdot \mathcal{H}(\Delta, B(K)) \subset \subset \text{Ker } \tilde{\pi}$. Therefore, because π is surjection, there exists a unique

$\phi : \mathcal{O}(S \times V) \rightarrow \mathcal{H}(S, B(K))$, such that the diagram

$$\begin{array}{ccc} \mathcal{O}(\Delta \times V) & \xrightarrow{\phi} & \mathcal{H}(\Delta, B(K)) \\ \pi \downarrow & & \downarrow \tilde{\pi} \\ \mathcal{O}(S \times V) & \xrightarrow{\phi} & \mathcal{H}(S, B(K)) \end{array}$$

is commutative; ϕ is evidently an \mathcal{O}_S -algebra homomorphism.

§ 2. The flatness and privilege theorem

Notation

Let S be an analytic space, U an open set in \mathbf{C}^n , and $\pi : S \times U \rightarrow S$ the first projection.

If \mathcal{F} is an $\mathcal{O}_{S \times U}$ module, then for every $s \in S$ we denote by $\mathcal{F}(s)$ the \mathcal{O}_U -module $i_s^* \mathcal{F}$, where i_s is the injective morphism $x \rightarrow (s, x)$ from U into $S \times U$. If $x \in U$

$$(\mathcal{F}(s))_x \simeq \mathcal{F}_{(s, x)} / m_s \cdot \mathcal{F}_{(s, x)} \simeq \mathcal{F}_{(s, x)} \otimes_{\mathcal{O}_{S, s}} \mathbf{C}_s.$$

Theorem 1: Let \mathcal{E} be a coherent and S -flat $\mathcal{O}_{S \times U}$ -module, and K a poly-cylinder in U .

(a) When K is privileged for $\mathcal{E}(s_0)$, s_0 has a neighbourhood V such that K is $\mathcal{E}(s)$ -privileged for each $s \in V$. In other words: the set $S' = \{s \in S \mid K \text{ is } \mathcal{E}(s)\text{-privileged}\}$ is open in S .

(b) It is possible to define a Banach vector bundle over S' whose fibre at any $s \in S'$ is $B(K, \mathcal{E}(s))$.

To prove the theorem we need:

Lemma 1: Under the conditions of the theorem, we can, for every $s \in S$, find a neighbourhood W of $\{s\} \times K$ and a free resolution of finite length

$$0 \rightarrow \mathcal{L}_p \xrightarrow{d_p} \dots \xrightarrow{d_2} \mathcal{L}_1 \xrightarrow{d_1} \mathcal{L}_0 \xrightarrow{\varepsilon} \mathcal{E} \rightarrow 0 \text{ in } W.$$

Proof: Let (s, x) be a point of $S \times U$ and \mathcal{L}_*^0 a finite resolution of $\mathcal{F}(x)$ in a neighbourhood of x (there exists such one, by the theorem of syzygies). We shall show that there exists a resolution \mathcal{L}^* of \mathcal{F} in a neighbourhood of (s, x) such that $\mathcal{L}^*(s) = \mathcal{L}_*^0$; if $\mathcal{L}_i^0 = \mathcal{O}_x^{r_i}$ define

$$\mathcal{L}_i = \mathcal{O}_{S \times U}^{r_i} \text{ and } \mathcal{K}_i^0 = \text{Ker } d_i^0 : \mathcal{L}_i^0 \rightarrow \mathcal{L}_{i-1}^0.$$