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*Remark :* This a particular case of the following proposition: if  $\pi$  and  $\pi'$  are two morphisms of which at least one is finite, then

$$\begin{array}{ccc} X & & Y \\ \pi \searrow & S & \swarrow \pi' \\ & S & \end{array}, \quad \mathcal{O}_{S \times Y} = \mathcal{O}_X \otimes_{\mathcal{O}_S} \mathcal{O}_Y.$$

We have proved that  $\mathcal{O}_{W \times X}$  is  $\mathcal{O}_W$ -flat, so by scalar extension  $\mathcal{O}_{S \times X}$  is  $\mathcal{O}_S$  flat.

*Corollary :* If  $X$  and  $S$  are two manifolds and  $\pi : X \rightarrow S$  is a submersion, then  $\pi$  is flat.

### III. PRIVILEGED POLYCYLINDERS

#### § 1. *Banach vector bundles over an analytic space*

Let  $E$  be a Banach space and  $X$  an analytic space. We denote then by  $E_X$  the trivial bundle  $X \times E$  over  $X$ .

To define bundle morphisms, we first define the sheaf  $\mathcal{H}_X(E)$  of germs of analytic morphisms from  $X$  to  $E$ . If  $U \subset \mathbb{C}^n$  is open, then the set  $\mathcal{H}(U, E)$  of analytic morphisms from  $U$  into  $E$  consists of all functions  $g : U \rightarrow E$  having at every point  $x \in U$  a converging power series expansion.

Let now  $X'$  be a local model for  $X$ , i.e.  $X'$  is the support of the quotient sheaf  $\mathcal{O}_U/J$ , where  $U \subset \mathbb{C}^n$  is open and  $J$  is a coherent sheaf of ideals of  $\mathcal{O}_U$ , then  $\mathcal{H}_{X'}(E)$  is the sheaf associated to the presheaf  $V \mapsto \mathcal{H}(V, E)/J_V \cdot \mathcal{H}(V, E)$  ( $V \subset U$ ,  $V$ -open).

*Remark :* If  $X'$  is reduced, the sections of  $\mathcal{H}_{X'}(E)$  are just the functions from  $X'$  to  $E$  which are locally induced by analytic functions on open sets in  $U$ .

The sheaf  $\mathcal{H}_X(E)$  is constructed with help of the local models  $X'$  of  $X$ , i.e.  $\mathcal{H}_X(E)|X' = \mathcal{H}_{X'}(E)$ , for every local model  $X'$ .

*Definition 1 :* The set of *analytic morphisms* from an analytic space  $X$  into a Banach space  $E$  is the set  $\mathcal{H}(X; E)$  of sections of the sheaf  $\mathcal{H}_X(E)$ .

Let  $\mathcal{L}(E, F)$  be the Banach space of all continuous linear mappings from the Banach space  $E$  into the Banach space  $F$ .

*Definition 2 :* An analytic vector bundle morphism from  $E_X$  into  $F_X$  is an analytic morphism from  $X$  into  $\mathcal{L}(E, F)$ .

Let  $E$  be a topological space,  $X$  an analytic space, and  $\pi : E \rightarrow X$  a continuous projection.

$$\begin{array}{ccccc}
 & E & \xleftarrow{\quad} & E|_{U_i} & \xrightarrow{\quad \phi_i \quad} E_{iU_i} \\
 \pi \downarrow & & \searrow & & \swarrow \\
 X & \xleftarrow{\quad} & U_i & \xrightarrow{\quad} & 
 \end{array}$$

Suppose that  $X$  has an open covering  $(U_i)_{i \in I}$ , and that for every  $i \in I$  there is given a trivial Banach space bundle  $E_{iU_i}$  and a homeomorphism  $\phi_i$ , such that the following diagram is commutative:

We suppose further that for each pair  $i, \kappa \in I$  there is given an analytic vector bundle morphism  $\gamma_{i\kappa} : E_{\kappa U_i \cap U_\kappa} \rightarrow E_{iU_i \cap U_\kappa}$ , with the underlying mapping  $\phi_i \circ \phi_\kappa^{-1}$ , such that:

$$\gamma_{i\lambda} = \gamma_{i\kappa} \gamma_{\kappa\lambda}; \quad \gamma_{i\lambda} = I, \quad \text{for all } i, \kappa, \lambda \in I.$$

This data gives a Banach vector bundle atlas on  $E$  and provides  $E$  with the structure of a Banach vector bundle over  $X$  (two atlases are equivalent if there exists an atlas containing both).

*Remark:* If  $X$  is reduced, the  $\gamma_{i\kappa}$  are determined by their underlying map and the condition  $\gamma_{i\lambda} = \gamma_{i\kappa} \gamma_{\kappa\lambda}$  is automatically satisfied.

Using local triviality, we can define morphisms for general Banach vector bundles.

*Proposition 1:* Let  $\phi : E \rightarrow F$  be a morphism of two Banach vector

$$\begin{array}{c}
 \swarrow \quad \searrow \\
 X
 \end{array}$$

bundles  $E$  and  $F$ , and  $x \in X$ .

If  $\phi_x \in \mathcal{L}(E(x), F(x))$  is an isomorphism, then there exists an open neighbourhood  $U \subset X$  of  $x$ , such that  $\phi|U : E|U \rightarrow F|U$  is a vector bundle isomorphism.

*Proof:* First we take a trivialisation  $E|V = E_0|V$ ,  $F|V = F_0|V$  at  $x \in V \subset X$  ( $V$ -open).

The set  $\text{Isom}(E_0, F_0)$  of isomorphic mappings is an open subset of  $\mathcal{L}(E_0, F_0)$  and the mapping  $g \rightarrow g^{-1}$  is an analytic isomorphism:

$$\text{Isom}(E_0, F_0) \simeq \text{Isom}(F_0, E_0).$$

So we have in an open neighbourhood  $U \subset X$  of  $x$  an analytic morphism  $y \rightarrow \phi_y^{-1} \in \mathcal{L}(F_0, E_0)$ , which defines the inverse morphism  $(\phi|U)^{-1} : F|U \rightarrow E|U$ .

*Definition 3 :* Let  $E$  and  $F$  be two Banach spaces and  $f$  a continuous linear mapping from  $E$  into  $F$ .  $f$  is a *split mono-(epi) morphism*, if there exists a mapping  $g \in \mathcal{L}(F, E)$  such that  $g \circ f = I_E$ . (Resp.  $f \circ g = I_F$ .)

*Definition 4 :* Let  $E_1$  and  $E_2$  be two Banach vector bundles over an analytic space  $X$ , and  $f$  a vector bundle morphism from  $E_1$  into  $E_2$ .  $f$  is a *split mono (epi) morphism*, if there exists a vector bundle morphism  $g : E_2 \rightarrow E_1$  such that  $g \circ f = I_{E_1}$ . (Resp.  $f \circ g = I_{E_2}$ .)

Equivalently,  $f : E_1 \rightarrow E_2$  is a split monomorphism if and only if  $E_2$  can

$$\begin{array}{ccc} & & \\ & \searrow & \swarrow \\ & X & \end{array}$$

be decomposed in a direct sum  $E_2 = F_2 \oplus G_2$  such that

$$f : \begin{cases} E_1 \simeq F_2 \\ 0 \rightarrow G_2 \end{cases} .$$

and  $f$  is a split epimorphism if correspondingly

$$E_1 = F_1 \oplus G_1, \quad \text{such that} \quad f : \begin{cases} F_1 \rightarrow 0 \\ G_1 \simeq E_2 \end{cases} .$$

*Proposition 2 :* Let  $E \xrightarrow{\varphi} F$  be a bundle morphism and  $x \in X$ .

$$\begin{array}{ccc} & & \\ & \searrow & \swarrow \\ & X & \end{array}$$

If  $\phi_x : E(x) \rightarrow F(x)$  is a split epi (mono) morphism, then the point  $x$  has an open neighbourhood  $U \subset X$ , such that  $\phi|U : E|U \rightarrow F|U$  is a split vector bundle epi (mono) morphism.

*Proof :* Suppose that  $\phi_x$  is a split epimorphism. We take first a trivialisation  $E|V = E_{0V}$ ,  $F|V = F_{0V}$  at  $x$ , so that there exists a mapping  $\sigma \in \mathcal{L}(F_0, E_0)$ ,  $\phi_x \circ \sigma = I_{F_0}$ . If we define a morphism  $\psi : F_{0V} \rightarrow E_{0V}$  by  $x \rightarrow \sigma \in \mathcal{L}(F_0, E_0)$ , the morphism  $\gamma = \phi \circ \psi : F_{0V} \rightarrow F_{0V}$  has an isomorphic fibre mapping  $\gamma_x = I_{F_{0V}}$  in  $x$ . By proposition 1 we have an isomorphic restriction  $\gamma|U$ ,  $\phi|U \circ (\psi|U \circ (\gamma|U)^{-1}) = I_{F_{0U}}$ .

When  $\phi_x$  is a split monomorphism, the proof is similar.

*Definition 5 :* Let  $B_1, B_2, B_3$  be Banach spaces, and  $j, k : B_1 \xrightarrow{j} B_2 \xrightarrow{k} B_3$  continuous linear mappings. This sequence forms a *complex*, if  $k \circ j = 0$ . This sequence is *split exact* if the space  $B_i$  can be decomposed in direct

sums  $B_i = C_i \oplus D_i$  such that

$$j : \begin{cases} C_1 \rightarrow 0 \\ D_1 \simeq C_2 \end{cases} \quad k : \begin{cases} C_2 \rightarrow 0 \\ D_2 \simeq C_3 \end{cases} .$$

*Definition 6:* A Banach vector bundle morphism sequence

$$\begin{array}{ccccc} E_1 & \xrightarrow{f} & E_2 & \xrightarrow{g} & E_3 \\ & \searrow & \downarrow & \swarrow & \\ & & x & & \end{array} \quad \text{is a complex if } g \circ f = 0.$$

The sequence is *split exact*, if every  $E_i$  can be decomposed  $E_i = F_i \oplus G_i$ , such that:

$$f : \begin{cases} F_1 \rightarrow 0 \\ G_1 \simeq F_2 \end{cases} \quad g : \begin{cases} F_2 \rightarrow 0 \\ G_2 \simeq F_3 \end{cases} .$$

*Theorem 1:* Let  $\begin{array}{ccccc} E_1 & \xrightarrow{f} & E_2 & \xrightarrow{g} & E_3 \\ & \searrow & \downarrow & \swarrow & \\ & & x & & \end{array}$  be a complex of Banach vector

bundles and  $x_0 \in X$ .

If the sequence of Banach spaces  $E_1(x_0) \xrightarrow{f_{x_0}} E_2(x_0) \xrightarrow{g_{x_0}} E_3(x_0)$  is split exact, then there exists an open neighbourhood  $U \subset X$  of  $x_0$ , such that  $E_1|U \rightarrow E_2|U \rightarrow E_3|U$  is a split exact sequence of Banach vector bundles.

*Proof:* We take a neighbourhood  $V$  of  $x$ , such that we have a complex  $E_{1V} \rightarrow E_{2V} \rightarrow E_{3V}$  of trivial bundles. By assumption we have the decompositions  $E_{iV}(x_0) = F_i(x_0) \oplus G_i(x_0)$  with

$$f_{x_0} : \begin{cases} F_1(x_0) \rightarrow 0 \\ G_1(x_0) \simeq F_2(x_0) \end{cases} \quad g_{x_0} : \begin{cases} F_2(x_0) \rightarrow 0 \\ G_2(x_0) \simeq F_3(x_0) \end{cases} .$$

By proposition 2,  $f|V : G_{1V} \rightarrow E_{2V}$ ,  $g|V : G_{2V} \rightarrow E_{3V}$  are both split monomorphisms in a neighbourhood  $W \subset V$  of  $x_0$  and the images  $F_2 = f(G_{1W})$ ,  $F_3 = g(G_{2W})$  are subbundles of  $E_{2W}$  esp.  $E_{3W}$ , such that

$$E_{2W} = F_2 \oplus G_{2W}, \quad E_{3W} = F_3 \oplus G_{3W}.$$

By our construction

$$g|_W : \begin{cases} F_2 \rightarrow 0 \\ G_2|_W \simeq F_3 \end{cases} .$$

If  $p: E_{2W} \rightarrow F_2$  is the projection with kernel  $G_{2W}$ , the map,  $p \circ f: E_{1W} \rightarrow F_2$  is a split epimorphism in  $x_0$ . Again by prop. 2 we have over an open neighbourhood  $U \subset W$  of  $x_0$  a decomposition  $E_{1U} = F_1 \oplus G_{1U}$  (with  $F_1 = \text{Ker } p \circ f$ )

$$(p \circ f)|_U : \begin{cases} F_1 \rightarrow 0 \\ G_{1U} \xrightarrow{\sim} F_{2U} \end{cases} .$$

The image  $f|_U(F_1)$  is contained in  $G_{2U}$ . But  $g|_U \circ f|_U = 0$  and  $g|_{G_{2U}}$  is a monomorphism hence  $f|_U: F_1 \rightarrow 0$ . We get finally (restricting all our morphisms to  $U$ )

$$f|_U : \begin{cases} F_{1U} \rightarrow 0 \\ G_{1U} \simeq F_{2U} \end{cases} \quad g|_U : \begin{cases} F_{2U} \rightarrow 0 \\ G_{2U} \xrightarrow{\sim} F_{3U} \end{cases} .$$

## § 2. Privileged polycylinders

*Definition 1:* A polycylinder in  $\mathbf{C}^n$  is a compact set  $K$  of the form  $K = K_1 \times \dots \times K_n$  where each  $K_i$  is a compact, convex subset of  $\mathbf{C}$ , with nonempty interior. If each  $K_i$  is a disc, then  $K$  is a polydisc. We first recall the following theorem of Cartan.

*Theorem 1:* Let  $K$  be a polycylinder contained in an open subset  $U$  of  $\mathbf{C}^n$ . Let  $\mathcal{F}$  be a coherent analytic sheaf on  $U$ .

(A) There exists an open neighbourhood of  $K$  over which  $\mathcal{F}$  admits a finite free resolution

$$0 \rightarrow \mathcal{L}_n \rightarrow \dots \rightarrow \mathcal{L}_1 \rightarrow \mathcal{L}_0 \rightarrow \mathcal{F} \rightarrow 0 .$$

(B)  $H^q(K, \mathcal{F}) = 0$  for  $q > 0$ .

(Reference: For instance Gunning and Rossi.)

We have the following consequences of this theorem:

1) Given a finite free resolution

$$0 \rightarrow \mathcal{L}_n \rightarrow \dots \rightarrow \mathcal{L}_1 \rightarrow \mathcal{L}_0 \rightarrow \mathcal{F} \rightarrow 0$$

of a coherent sheaf  $\mathcal{F}$ , the sequence

$$0 \rightarrow \mathcal{L}_n(K) \rightarrow \dots \rightarrow \mathcal{L}_0(K) \rightarrow \mathcal{F}(K) \rightarrow 0$$

is an  $\mathcal{O}_U(K)$  - free resolution of  $\mathcal{F}(K)$ .

2) Given a short exact sequence of coherent sheaves

$$0 \rightarrow \mathcal{F}' \rightarrow \mathcal{F} \rightarrow \mathcal{F}'' \rightarrow 0,$$

then the sequence

$$0 \rightarrow \mathcal{F}'(K) \rightarrow \mathcal{F}(K) \rightarrow \mathcal{F}''(K) \rightarrow 0 \quad \text{is exact.}$$

Let  $\mathcal{F}$  be a coherent analytic sheaf on  $U$ , and let  $K \subset U$  be a polycylinder. If  $V$  is an open neighbourhood of  $K$ , then  $\mathcal{F}(V)$  can be equipped with a Fréchet-space structure (see: Malgrange).

Hence we can give  $\mathcal{F}(K)$  the structure of inductive limit of Fréchet-spaces. It is however essential for certain purposes to have Banach-spaces. This can be obtained by choosing a space slightly different from  $\mathcal{F}(K)$  and by choosing  $K$  in a “privileged” way.

Let  $B(K) = \{f: K \rightarrow \mathbf{C} \mid f \text{ continuous on } K \text{ and analytic on } \overset{\circ}{K}\}$ , then  $B(K)$  is Banach algebra and  $B(K) \subset C(K)$ . The sections of  $\mathcal{O}_U$  over  $K$  are elements of  $B(K)$ , and  $B(K)$  is in fact the uniform closure of  $\mathcal{O}_U(K)$  in  $C(K)$ .

If  $\mathcal{L} = \mathcal{O}_U^r$ , we define  $B(K, \mathcal{L}) = B(K)^r$ . Then  $B(K; \mathcal{L})$  is a free  $B(K)$ -module, and since  $\mathcal{L}(K) = \mathcal{O}_U(K)^r$ , we have  $B(K; \mathcal{L}) = B(K) \otimes_{\mathcal{O}_U(K)} \mathcal{L}(K)$ .

We now assume that  $\mathcal{F}$  is a coherent sheaf on  $U$ , where  $U \subset \mathbf{C}^n$  is open. Consider a free resolution

$$(R) \quad 0 \rightarrow \mathcal{L}_n \rightarrow \dots \rightarrow \mathcal{L}_1 \rightarrow \mathcal{L}_0 \rightarrow \mathcal{F} \rightarrow 0 \quad \text{of } \mathcal{F}.$$

From  $(R)$  we get an  $\mathcal{O}_U(K)$ -free resolution of  $\mathcal{F}(K)$

$$(R') \quad 0 \rightarrow \mathcal{L}_n(K) \rightarrow \dots \rightarrow \mathcal{L}_1(K) \rightarrow \mathcal{L}_0(K) \rightarrow \mathcal{F}(K) \rightarrow 0.$$

Taking the tensorproduct  $B(K) \otimes_{\mathcal{O}_U(K)} \mathcal{L}$  we get the complex

$$B(K; \mathcal{L}): 0 \rightarrow B(K; \mathcal{L}_n) \rightarrow \dots \rightarrow B(K; \mathcal{L}_1) \rightarrow B(K; \mathcal{L}_0).$$

*Definition 2:* The polycylinder  $K$  is called  $\mathcal{F}$ -privileged if the complex  $B(K; \mathcal{L})$  is split-exact in every degree  $> 0$ .

*Remark:* The property of being  $\mathcal{F}$ -privileged is independent of the resolution  $(R)$ .

The exactness of  $B(K; \mathcal{L})$  can be expressed by  $\text{Tor}_i^{\mathcal{O}(K)}(B(K), \mathcal{F}(K)) = 0$ , for every  $i > 0$ , and  $\text{Tor}$  is independent of the resolution  $(R)$ . It is a little

more complicated to show, that the splitting property is independent of  $(R)$ , and this is omitted.

Since  $B(K; \mathcal{L}_i)$  is a Banach space, the image and its complement are thus Banach spaces if  $K$  is  $\mathcal{F}$ -privileged. In this case we define  $B(K; \mathcal{F}) = \text{Coker } (B(K, \mathcal{L}_1) \rightarrow B(K, \mathcal{L}_0)) = B(K) \otimes_{\mathcal{O}_U} \mathcal{F}(K)$  and we get a  $B(K)$ -module, which is a Banach-space.

*Warning* : In the definition of split-exactness, the subspaces are splitting vector spaces, but they are not splitting  $B(K)$ -modules in general.

We have the following important theorem about the existence of privileged polycylinders:

*Theorem 2* : Let  $U$  be an open subset of  $\mathbf{C}^n$ , and let  $\mathcal{F}$  be a coherent analytic sheaf on  $U$ . For any  $x \in U$  there exists a fundamental system of neighbourhoods of  $x$  in  $U$ , which are  $\mathcal{F}$ -privileged polycylinders.

For the proof, see Douady: § 7, 4, th 1.

*Example* : (Curves in  $\mathbf{C}^2$ ) Let  $U \subset \mathbf{C}^2$  be an open connected neighbourhood of the origin, and let  $h: U \rightarrow \mathbf{C}$  be analytic and  $h \neq 0$ .

Let  $X$  be the curve given by  $h$ , that is  $X = h^{-1}(0)$ ,  $\mathcal{O}_X = \mathcal{O}_U/(h)$ . We have an exact sequence  $0 \rightarrow \mathcal{O}_U \rightarrow \mathcal{O}_U \rightarrow \mathcal{O}_X \rightarrow 0$ . Consider a polycylinder  $K = K_1 \times K_2 \subset U$ . By definition  $K$  is  $\mathcal{O}_X$ -privileged if and only if  $h: B(K) \rightarrow B(K)$  is a split monomorphism.

Let  $K_j$  denote the boundary of  $K_j$ , and define  $\bar{K} = \bar{K}_1 \times \bar{K}_2$  ( $\bar{K}$  is called the Šilov Boundary of  $K$ ).

*Proposition 1* : (a) The following conditions are equivalent:

- (i)  $h: B(K) \rightarrow B(K)$  is a monomorphism.
- (i')  $\exists a > 0$  such that  $\|hf\| \geq a\|f\|$ ,  $\forall f \in B(K)$ .
- (ii)  $X \cap \bar{K} = \emptyset$ .

(b) If  $(K_1 \times K_2) \cap X = \emptyset$ , then  $h$  is a split monomorphism (i.e.  $K$  is  $\mathcal{O}_X$  privileged).

*Proof* : (a) (i)  $\Leftrightarrow$  (i') is a well known fact from the theory of normed vector spaces.

(ii)  $\Rightarrow$  (i'). Assume  $X \cap \bar{K} = \emptyset$ . If  $f \in B(K)$ , then it follows from the maximum principle that  $\|f\| = \sup_K |f(x)| = \sup_{\bar{K}} |f(x)|$ . Since  $h(x) \neq 0$

whenever  $x \in \overset{\circ}{K}$ , we get  $a = \inf_K |h(x)| > 0$ . Hence  $\|hf\| = \sup_K |hf(x)| \geq \geq a \sup_K |f(x)| = a \|f\|$ .

(i')  $\Rightarrow$  (ii). Suppose that  $X \cap \overset{\circ}{K} \neq \emptyset$  and  $x = (x_1, x_2) \in X \cap \overset{\circ}{K}$ . We choose an analytic function  $f_1 : U_1 \rightarrow \mathbf{C}$ , where  $U_1 \supset K_1$ , and  $U_1$  is open, such that  $f_1(x_1) = 1$ ,  $|f_1(z)| < 1$  if  $z \in K_1$ ,  $z \neq x_1$ . Similarly we choose an analytic function  $f_2 : U_2 \rightarrow \mathbf{C}$ , with the same properties. Consider the function  $f \in B(K) : (z_1, z_2) \rightarrow f_1(z_1)f_2(z_2)$ . Since  $h(x) = 0$  it follows that the sequence  $\{hf^n\}$  converges pointwise to 0 in  $K$ .

Applying Dini's theorem we get  $\|hf^n\| \rightarrow 0$ . From the inequality  $a \|f^n\| \leq \|hf^n\|$  we get  $\|f^n\| \rightarrow 0$ , which is a contradiction, because for every  $n : f^n(x) = 1$ .

(b) Use the Weierstrass preparation theorem (extended form).

*Question.* Does the condition (ii) imply that  $h : B(K) \rightarrow B(K)$  is a split monomorphism?

#### IV. FLATNESS AND PRIVILEGE

##### § 1. *Morphisms from an analytic space into $B(K)$*

Let  $S$  be an analytic space and  $K$  a polycylinder in an open set  $U \subset \mathbf{C}^n$ . We want to construct an  $\mathcal{O}_S$ -algebra homomorphism  $\phi : \mathcal{O}_{S \times U}(S \times U) \rightarrow \mathcal{H}(S; B(K))$ .

(a) Consider first  $S = U' \subset \mathbf{C}^m$ ,  $U'$ -open. If  $h \in \mathcal{O}_{U' \times U}(U' \times U)$  and  $s \in U'$ ,  $x \in K$ , define  $(\phi(h)(s))(x) = h(s, x)$ . Using the Cauchy integral, one can show that  $\phi(h)$  is analytic. On the other hand it is obvious that  $\phi$  is an  $\mathcal{O}_{U'}$ -algebra homomorphism.

(b) Let  $S$  have a special model in the polydisc  $\Delta$  in  $\mathbf{C}^m$ , defined by a sheaf  $\mathcal{J}$  of ideals of  $\mathcal{O}_\Delta$ , and let  $\mathcal{J}$  be generated by  $f_1, \dots, f_p$ ,  $V$ -a polycylinder neighbourhood of  $K$  in  $U$ . By Cartan's theorem  $B$  for a polycylinder,

the sequence  $0 \rightarrow \mathcal{J}(\Delta \times V) \xrightarrow{\pi} \mathcal{O}(\Delta \times V) \rightarrow \mathcal{O}(S \times V) \rightarrow 0$  is exact. If we denote by  $\tilde{\pi}$  the projection  $\mathcal{H}(\Delta, B(K)) \rightarrow \mathcal{H}(S, B(K))$ ,  $(f_1, \dots, f_p) \cdot \mathcal{H}(\Delta, B(K)) \subset \subset \text{Ker } \tilde{\pi}$ . Therefore, because  $\pi$  is surjection, there exists a unique

$\phi : \mathcal{O}(S \times V) \rightarrow \mathcal{H}(S, B(K))$ , such that the diagram