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**Autor:** Stein, K.  
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*Proposition 5.* If  $f : X \xrightarrow{k} Y$  is continuous and  $Y$  is a Hausdorff space,  $G_f$  is closed in  $X \times Y$ .

*Definition 2.* A correspondence  $f$  is *proper* if  $f$  and  $f^{-1}$  are continuous.<sup>1)</sup>

*Proposition 6.* If  $f : X \xrightarrow{k} Y$ ,  $f_1 : X_1 \xrightarrow{k} Y_1$ ,  $g : Y \xrightarrow{k} Z$  are proper, then  $f \times f_1$  and  $g \circ f$  are proper.

The junction of two proper correspondences need not, however, be proper. The diagonal mapping  $(I_X, I_X)$  serves as an example if  $X$  is not a Hausdorff space. If  $X$  is Hausdorff, the junction  $(f, f')$  of proper correspondences  $f : X \xrightarrow{k} Y$  and  $f' : X \xrightarrow{k} Y'$  remains proper.

*Proposition 7.* Let  $f : X \xrightarrow{k} Y$ ,  $f_1 : X \xrightarrow{k} Y_1$ ,  $g : Y \xrightarrow{k} Z$  be continuous where all the spaces are locally compact. Then we have:

- 1) If  $f$  is proper, then  $(f, f_1)$  and  $(f_1, f)$  are proper,
- 2) If  $g \circ f$  is proper and  $g^{-1}$  surjective, then  $f$  is proper,
- 3) If  $g \circ f$  is proper and  $f$  surjective, then  $g$  is proper.

## 2. HOLOMORPHIC CORRESPONDENCES

We consider reduced complex spaces  $(X, \theta)$  where  $X$  is assumed Hausdorff and where the structure sheaf  $\theta$  has no nilpotent elements. For the definition and related concepts we refer to [8]. The structure sheaf is usually omitted in the notation.

*Definition 3.* Let  $X$  and  $Y$  be complex spaces. A correspondence  $f : X \xrightarrow{k} Y$  is called *holomorphic* if

- 1)  $f$  is continuous,
- 2) the graph  $G_f$  is an analytic set in  $X \times Y$ .

If only the condition 2) is fulfilled,  $f$  is said to be *weakly holomorphic*.

Let  $f : X \xrightarrow{k} Y$  be weakly holomorphic. Then  $f^{-1}$  is weakly holomorphic; furthermore, if  $A \subset X$  is analytic in  $X$ ,  $f|A$  is weakly holomorphic. Since  $\check{f}^{-1}(x) = G_f \cap (\{x\} \times Y)$ ,  $x \in X$ , is analytic in  $G_f$ ,  $f(x) = \hat{f}(\check{f}^{-1}(x))$

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<sup>1)</sup> Compare [3] where another notion of proper correspondence is defined.

is analytic in  $Y$ . If  $f$  is holomorphic and  $A' \subset Y$  analytic in  $Y$ , then, since  $\hat{f}^{-1}(A')$  is analytic in  $G_f$  and  $\check{f}$  is proper,  $f^{-1}(A') = \check{f}(\hat{f}^{-1}(A'))$  is analytic in  $X$  by Remmert's mapping theorem [11] (see also [8], p. 129).

The correspondences  $f \times f_1$ ,  $(f, f'_1)$ , and  $g \circ f$  are holomorphic if the correspondences  $f, f_1, f'_1$ , and  $g$  are holomorphic.

A weakly holomorphic correspondence  $f: X \xrightarrow{k} Y$  is called *reducible* resp. *irreducible* if  $G_f$  is reducible resp. irreducible.  $G_f$  is always a union of irreducible components  $G^{(i)}$ ; let  $f_i: X \xrightarrow{k} Y$  be the (weakly holomorphic) correspondence whose graph is  $G^{(i)}$ . Then the correspondences  $f_i$  are called the irreducible components of  $f$  and we write  $f = \cup f_i$ .

### 3. MEROMORPHIC MAPPINGS

Let  $f: X \xrightarrow{k} Y$  be a correspondence where  $X$  is a topological space. A point  $x \in X$  is called a *distinguished point of  $f$*  if there is a neighborhood  $U$  of  $x$  such that the restriction  $f|U$  is a mapping (in the usual sense).

*Definition 4.* A holomorphic correspondence  $f: X \xrightarrow{k} Y$  is called a *meromorphic mapping* if the following holds. If  $X$  is irreducible, then

- 1)  $f$  is irreducible,
- 2) There exists a distinguished point  $x_0 \in X$  of  $f$ .

In the general case, if  $X = \cup X^{(i)}$  is the decomposition of  $X$  into irreducible components, then there exist holomorphic correspondences  $f_i: X \xrightarrow{k} Y$  such that

- 1)  $f_i|X^{(i)}$  is a meromorphic mapping and  $f_i|X - X^{(i)}$  is empty,
- 2)  $f = \cup f_i$ .

A meromorphic mapping  $f$  is *bimeromorphic* if  $f^{-1}$  is meromorphic.

We use the notation  $f: X \xrightarrow{m} Y$  for a meromorphic mapping. Note that a meromorphic mapping is in general not a mapping in the strong sense.

An example of a meromorphic mapping is the correspondence  $f$  of  $\mathbf{C}^2$  onto the extended complex plane  $\mathbf{P}_1$  defined by  $f(z_1, z_2) = \frac{z_1}{z_2}$  if  $(z_1, z_2) \neq (0, 0)$ , and  $f(0, 0) = \mathbf{P}_1$ .