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Theorem (Krull). Let A be a local noetherian ring, E a finite module over A. Then:

(i) The Krull topology of E is separated.

(ii) Every submodule F of E is closed in E.

(iii) The topology induced by E in a submodule F is the Krull topology of F.

Proof. (i) Let $F = \bigcap_{k \geq 0} \mathfrak{m}^k E = \overline{\{0\}}$. Then

$$\mathfrak{m}F = \mathfrak{m}\big((\mathfrak{m}^n E) \cap F\big) = (\mathfrak{m}^{n+1} E) \cap F = F$$

by the Artin-Rees lemma. Hence Nakayama's lemma implies that $F = \{ 0 \}$.

(ii) Let $f : E \to E/F$ be the natural map. Then

$$f(\bar{F}) \subset f(F + \mathfrak{m}^k E) = f(\mathfrak{m}^k E) = \mathfrak{m}^k (E/F) .$$

Hence $f(\bar{F}) \subset \cap\, \mathfrak{m}^k (E/F) = \{ 0 \}$, using (i).). But $f(\bar{F}) \subset \{ 0 \}$ is equivalent to $\bar{F} \subset F$.

(iii) It is clear that $\mathfrak{m}^k F \subset (\mathfrak{m}^k E) \cap F$. Hence the Krull topology of F is finer than that induced by E ; in other words the inclusion $F \to E$ is continuous. Conversely the Artin-Rees lemma shows that

$$(\mathfrak{m}^{n+k} E) \cap F = \mathfrak{m}^k \big((\mathfrak{m}^n E) \cap F\big) \subset \mathfrak{m}^k F$$

which proves that the induced topology is finer than the Krull topology of F.

REFERENCES

[1] Bourbaki, N. *Algèbre commutative*, ch. 3, 4. Hermann, Paris 1961.

[2] Grauert, H. Ein Theorem der analytischen Garbentheorie und die Modulräume komplexer Structuren, publications mathématiques, *I.H.E.S.*, n° 5, Paris 1960.

[3] —— *Lectures at Otaniemi* Finland 1967 (in this volume).

[4] Grothendieck, A. *Exposés 7-17 in Séminaire Cartan* Paris 1960/61.

[5] Gunning, R. C. and H. Rossi, *Analytic functions of several complex variables.* Prentice Hall, Englewood Cliffs, N.J. 1965.

[6] Houzel, C. *Exposés 18-21 in Séminaire Cartan*, Paris 1960/61.

[7] Mather, J. N. *Stability of C^∞ mappings II* (to be published in *Annals of Math.*).

[8] Nagata, M. Local rings. *Interscience*, New-York 1962.

[9] Narasimhan, R. Introduction to the theory of analytic spaces. *Lecture notes in Mathematics*, n° 25. Springer, Berlin 1966.