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Remark. If X is not separated, an intersection of two open Stein subspaces of X need not be Stein; take f.i. for X two copies of \mathbb{C}^2 , identified everywhere except at O ; there is an obvious covering of X by two open subspaces, identicals with \mathbb{C}^2 ; but their intersection is $\mathbb{C}^2 - \{O\}$, and therefore is not Stein!

4.4. The finiteness theorem

Theorem 4.4.1. (Cartan — Serre). Let X be a compact analytic space, and F be a coherent analytic sheaf on X . Then, for every $p \geq 0$ $H^p(X, F)$ is separated and finite dimensional.

We shall give two proofs of this theorem ; both are interesting for further applications.

1st proof. Let $\{X_i\}$ and $\{X'_i\}$ be two finite coverings of X of the type considered in the previous articles, such that, for every i , X'_i is relatively compact in X_i . Then, if we denote by \mathcal{U} (resp. \mathcal{U}') the covering $\{X_i\}$ (resp. $\{X'_i\}$), the natural restriction map $C^p(\mathcal{U}, F) \rightarrow C^p(\mathcal{U}', F)$ is compact.

Consider now the map

$$(\rho, d) : Z^p(\mathcal{U}, F) \oplus C^{p-1}(\mathcal{U}', F) \rightarrow Z^p(\mathcal{U}', F)$$

this map is surjective, and we have $(, d\rho) = (\rho, 0) + (0, d)$, $(\rho, 0)$ being compact ; then the following lemma proves that $\text{Im}(0, d)$ is closed and finite codimensional, q.e.d.

Lemma 4.4.2. Let E and F two Frechet spaces, u_1 and u_2 two linear continuous maps $E \rightarrow F$ such that $u_1 + u_2$ is surjective, and u_1 compact. Then $\text{Im}(u_2)$ is closed and finite codimensional. For the proof, see e.g. [5].

2nd proof. Consider \mathcal{U} and \mathcal{U}' as above, and consider the map $(\rho, d) : C^{p-1}(\mathcal{U}, F)/Z^{p-1}(\mathcal{U}, F) \rightarrow [C^{p-1}(\mathcal{U}', F)/Z^{p-1}(\mathcal{U}', F)] \oplus Z^p(\mathcal{U}, F)$ (ρ, d) is clearly injective. I claim that its image is closed: In fact, since $\bar{\rho} : H^p(\mathcal{U}, F) \rightarrow H^p(\mathcal{U}', F)$ is injective, this image consists of the pairs (\bar{a}', b) , $a' \in C^{p-1}(\mathcal{U}', F)$, $b \in Z^p(\mathcal{U}, F)$ such that $da' = \rho b$, which proves the assertion.

Now we have $(\rho, d) = (\rho, 0) + (0, d)$ and $(\rho, 0)$ is compact. By a well-known lemma, it results that $\text{Im}(0, d)$ is closed, which means that $H^p(\mathcal{U}, F)$ is separated.

Finally, since $\bar{\rho}$ is compact, and is an isomorphism, it follows that the identity map of $H^p(\mathcal{U}, F)$ into itself is compact ; therefore this space is finite dimensional ; this proves the theorem.