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Artikel: BOUNDEDNESS THEOREMS FOR SOLUTIONS OF u" (t) + a(t)f{u) g(u') =0 (IV)

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Theorem 6. Suppose that assumptions A_1, A_2, A_3 and A_4 hold and in addition that $a(t) > 0$ and $a'(t) \geqq 0$ for $t \geqq T$; then all solutions of (3.3) are bounded.

Proof. Integrate (3.3) in the following manner:

$$G(u'(t)) - G(u'(t_0)) + a(t) F(u(t)) - a(t_0) F(u(t_0)) = \int_{t_0}^{t} a'(s) F(u(s)) ds + \int_{t_0}^{t} \frac{h(t, u, u') u'(s) ds}{g(u')} \qquad (3.4)$$

where $G(v) = \int_0^{\gamma} \frac{s\,ds}{g(s)}$ and $F(u) = \int_0^u f(s)\,ds$. Taking absolute values and noting that $G(v) \geqq 0$ and $F(u) \geqq 0$, we obtain

$$a(t) F(u(t)) \leqq c_0 + c_1 + \int_{t_0}^{t} a'(s) F(u(s)) ds \qquad (3.5)$$

where $c_0 = G(u'(t_0)) + a(t_0) F(u(t_0\ O)$ and $c_1 = \int_{t_0}^{\infty} \gamma(s)\,ds$ are non-negative constants. From (3.5) and A_4 it is now clear that every solution of (3.3) are bounded (cf. [1]).

Corollary. In addition to the hypothesis of Theorem 6, suppose that assumption A_5 also holds and that $\lim_{t \to \infty} a(t) = k > 0$; then all solutions of (3.3) and their derivatives are bounded.

We note that by setting $h(t, u, u') \equiv 0$, the above result again reduces to Theorem 1 and its corollary. Other comparison theorems may be formulated in a similar way as Theorem 6 by extending the corresponding result for the homogeneous equation. Since the procedure is clear, the statements and proofs of these results will be omitted.

RÉFÉRENCES

[1] J. S. W. Wong, A note on boundedness theorems of certain second order differential equations. *SIAM Review*, 6 (1964), 175-176.

[2] —— and T. A. Burton, Some properties of solutions of $u''(t) + a(t) f(u) g(u') = 0$ (II). *Monatshefte für Mathematik*, 69 (1965), 368-374.

[3] J. S. W. Wong, Some properties of solutions of $u''(t) + a(t) f(u) g(u) = 0$ (III). *SIAM Journal*, 14 (1966), 209-214.

[4] Zhang Bing-gen (Chang Ping-ken), Boundedness of solutions of ordinary differential equations of the second order. *Acta Math. Sinica*, 14 (1964), 128-137; English translation, *Chinese Mathematics*, 4 (1964), 139-148.

[5] C. T. TAAM, Criteria of boundedness of the solutions of non-linear differential equations. *Proc. Amer. Math. Soc.*, 6 (1955), 377-385.

[6] I. BIHARI, Researches of the boundedness and stability of the solutions of non-linear differential equations, *Acta. Math. Acad. Sci. Hungar.*, 8 (1957), 261-278.

[7] W. R. UTZ, Properties of solutions of $u'' + g(t)\, u^{2n-1} = 0$. *Monatshefte für Mathematik*, 66 (1962), 55-60.

[8] K. M. DAS, Properties of solutions of certain non-linear differential equations. *J. of Math. Ana. and Appl.*, 8 (1964), 445-452.

[9] C. M. PETTY and G. LEITMANN, A boundedness theorem for a non-linear, non-autonomous system. *Monatshefte für Mathematik*, 68 (1964), 46-51.

[10] R. BELLMAN, *Stability theorem of differential equations.* McGraw-Hill, New York (1953).

[11] I. BIHARI, A generalization of a lemma of Bellman and its application to uniqueness problems of differential equations. *Acta Math. Hung.*, 7 (1956), 81-94.

[12] E. F. BECKENBACH and R. BELLMAN, *Inequalities.* Springer-Verlag, Berlin (1961).

[13] Ju. A. KLOKOV, Some theorems on boundedness of solutions of ordinary differential equations. *Uspehi Mat. Nauk*, 13 (1958), No. 2 (80), 189-194; English translation. *Amer. Math. Soc. Transl.*, 18 (1961), 289-294.

[14] P. WALTMAN, Some properties of solutions of $u''(t) + a(t) f(u) = 0$. *Monatshefte für Mathematik*, 67 (1963), 50-54.

[15] LI YUE-SHENG (LI YUEH-SHENG), The bound, stability, and error estimates for the solution of non-linear differential equations. *Acta Math. Sinica*, 12 (1962), 32-39; English translation, *Chinese Mathematics*, 3 (1963), 34-41.

[16] D. WILLETT and J. S. W. WONG, On the discrete analogues of some generalizations of Gronwall's inequality. *Monatshefte für Mathematik*, 69 (1965), 362-367.

[17] J. S. W. WONG, On two theorems of Waltman. *SIAM Journal*, 14 (1966), 724-728.

[18] P. WALTMAN, On the asymptotic behaviour of solutions of a non-linear equation. *Proc. Amer. Math. Soc.*, 15 (1964), 918-923.

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