

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 13 (1967)  
**Heft:** 1: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** BOUNDEDNESS THEOREMS FOR SOLUTIONS OF  $u''(t) + a(t)f(u)g(u') = 0$  (IV)  
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**Bibliographie**  
**DOI:** <https://doi.org/10.5169/seals-41539>

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*Theorem 6.* Suppose that assumptions  $A_1, A_2, A_3$  and  $A_4$  hold and in addition that  $a(t) > 0$  and  $a'(t) \geq 0$  for  $t \geq T$ ; then all solutions of (3.3) are bounded.

*Proof.* Integrate (3.3) in the following manner:

$$G(u'(t)) - G(u'(t_0)) + a(t)F(u(t)) - a(t_0)F(u(t_0)) \\ = \int_{t_0}^t a'(s)F(u(s))ds + \int_{t_0}^t \frac{h(t, u, u')u'(s)ds}{g(u')} \quad (3.4)$$

where  $G(v) = \int_0^v \frac{s ds}{g(s)}$  and  $F(u) = \int_0^u f(s) ds$ . Taking absolute values and noting that  $G(v) \geq 0$  and  $F(u) \geq 0$ , we obtain

$$a(t)F(u(t)) \leq c_0 + c_1 + \int_{t_0}^t a'(s)F(u(s))ds \quad (3.5)$$

where  $c_0 = G(u'(t_0)) + a(t_0)F(u(t_0))$  and  $c_1 = \int_{t_0}^{\infty} \gamma(s) ds$  are non-negative constants. From (3.5) and  $A_4$  it is now clear that every solution of (3.3) are bounded (cf. [1]).

*Corollary.* In addition to the hypothesis of Theorem 6, suppose that assumption  $A_5$  also holds and that  $\lim_{t \rightarrow \infty} a(t) = k > 0$ ; then all solutions of (3.3) and their derivatives are bounded.

We note that by setting  $h(t, u, u') \equiv 0$ , the above result again reduces to Theorem 1 and its corollary. Other comparison theorems may be formulated in a similar way as Theorem 6 by extending the corresponding result for the homogeneous equation. Since the procedure is clear, the statements and proofs of these results will be omitted.

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(Reçu le 31 janvier 1967)

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