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for some positive constant k_2 in some neighborhood of infinity say, $\{u : |u| > \frac{1}{\varepsilon_2}, \varepsilon_2 > 0\}$, where p is the same fixed number as given in (14). Then (1) has no non-trivial L^{2p} solutions.

Proof. Let $x(t)$ be a non-trivial L^{2p} solution. Denote $A = \{t: |x(t)| < \varepsilon_1\}$, $B = \{t : |x(t)| > \frac{1}{\varepsilon_2}\}$ and $C = [0, \infty) - A - B$. Observe from (14) and (16) that

$$\int_0^\infty |\varphi(x(t))|^2 \, dt = \int_A + \int_B + \int_C$$

$$\leqq \int_A k_1 |x(t)|^{2p} dt + \frac{1}{\varepsilon_2} \mu(C) + \int_B k_2 |x(t)|^{2p} dt$$

$$< \infty ,$$

where $\mu(C)$ denotes the ordinary Lebesgue measure of C. Hence $x(t) \in L\Phi$. Following the proof of the main theorem, we again arrive at a contradiction, proving that (1) has no non-trivial L^{2p} solutions.

REMARK 1. Corollary 1 reduces to the result of Suyemoto and Waltman by taking $\varphi(u) = u^p$, $p \geqq 1$ which reduces to the result of Wintner when $p = 1$.

REMARK 2. Let $\varphi(u) = 2u + \sin u$. It is easy to see that $\varphi(u)$ satisfies all (7), (8), (14) and (16). Hence, from Corollary 2, we may conclude that equation (1) has no square integrable solutions.

REFERENCES

[1] SUYEMOTO, L. and P. WALTMAN, Extension of a theorem of A. Wintner. *Proc. Amer. Math, Soc.*, 14 (1963), 970-971.

[2] WINTNER, A., A criterion for the non-existence of L^2-solutions of a nonoscillatory diffreential equation. *J. London Math, Soc.*, 25 (1950), 347-351.

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Added in Proof. For a closely related paper, see J. Burlak, "On the non-existence of L² solutions of a class of non-linear differential equations", Proc. Edin. Math. Soc., 14(1965), 257-268, which contains several similar results.