Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 12 (1966)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SOME APPLICATIONS OF THE GAUSS-LUCAS THEOREM

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Bibliographie

DOI: https://doi.org/10.5169/seals-40726

Proof. It is enough to prove that f has a zero, and we may clearly suppose that $n_0 = 0$. From the proceding result, we see that the n_k-th partial sum of the power series for f has a zero in the disc

$$|z| \leq \left\{ \frac{|a_0|}{|a_1|} \frac{1}{\left(1 - \frac{n_1}{n_2}\right)\left(1 - \frac{n_1}{n_3}\right) \cdots \left(1 - \frac{n_1}{n_k}\right)} \right\}^{1/n_k} .$$

But since $\sum 1/n_k < \infty$, the product $\prod_2^\infty (1-(n_1/n_k))$ converges, so that there is a fixed disc with center at the origin that contains a zero of the n_k-th partial sum for $k = 2, 3, 4, \ldots$. It follows that f has a zero in this disc, and the result is proved.

It should be pointed out that Biernacki [1] proved, under the same hypotheses, and using a stronger form of the Gauss-Lucas Theorem, that f takes each complex value infinitely often. It is likely that our method can give a slight improvement of the preceding result, but not to the full strength of Biernacki's result. A recent result of G. and M. Weiss [5] gives a partial analogue for functions regular in the unit disc.

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(Reçu le 30 août 1964.)

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