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Artikel: ON ERMAKOF'S CONVERGENCE CRITERIA AND ABEL'S FUNCTIONAL EQUATION.

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Bibliographie

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THEOREM 1. *Assume that we have (4) for a sequence* $b_\nu \geqq \nu_0$ $(\nu = 1,2,\ldots)$. *Then, if we have (42) for almost all* $x \geqq \nu_0$ *and for a positive* $\alpha < 1$ *the series (18) is convergent.*

Further, assuming that $f(\nu)$ is not $= 0$ for all sufficiently great integers ν, we have for all $x \geqq \nu_0$:

THEOREM 2. *Assume that there exists an* $a \geqq \nu_0$ *and an integer* $\nu_1 \geqq \nu_0$ *such that:*

$$\Psi(a) > \nu_1 \geqq \psi(a)\,,\ f(\nu_1) > 0\,, \tag{44}$$

and a sequence $b_\nu \geqq \nu_0$ $(\nu = 1,2,\ldots)$ *such that we have (8). Then, if (43) holds for almost all* $x \geqq \nu_0$, *the series (18) is divergent and we have (10) for all* $x \geqq a$.

THEOREM 3. *Assume that there exists a constant* γ, $0 < \gamma < 1$, *and a sequence* $b_\nu \geqq \nu_0$ *such that (13) holds and further that for a constant c and for all integers* $\nu \geqq \nu_1$ *we have:*

$$\nu f(\nu) \leqq c \quad (\nu \geqq \nu_1)\,.$$

If then (42) holds for a certain $\alpha < 1$ *the series (18) is convergent, and the relation* $\Psi(a) \leqq \psi(a)$ *is for an* $a \geqq \nu_0$ *only possible, if* $f(\nu) = 0$ *for all* $\nu \geqq [\Psi(a)]$.

Observe that in applying the Theorems $1', 2'$ and $3'$ to φ the transformation formula can be certainly applied since $|\varphi(x)|$ is uniformly bounded.

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