Thesis 10. (Corollary to theses 8 and 9).

Proposals for the teaching of mathematics must include proposals for typical examinations, together with a statement of the kinds of examination results that will be considered acceptable *in terms of the success or failure of the proposals.*

I may add here that, if the aims include the mastery of *ideas* by the student (for instance an understanding for the idea of mathematical structure), then the examinations must include tests for mastery of ideas, that is essay-type questions; e.g. " Explain and discuss the idea of mathematical structure ". In a system of education like the French one, such questions might build valuable bridges between the teaching of mathematics and that of philosophy, particularly philosophy of science.

II. The Process of Reform

So far, I have been discussing basic principles that, in my opinion, ought to govern the process of elaborating, implementing, and assessing proposals for shaping the teaching of mathematics.

The most basic principle of all, however, has not been stated so far. This simply is that *proper care* must be exercised in shaping the teaching of mathematics. Everything else will be to no avail if such care is absent, if we allow sloppiness and irresponsibility to prevail; exactly as progress in medical science will be of no avail if medical practitioners fail to wash their hands and to learn about proper dosages of antibiotics. In the past, unfortunately, proper care has not always been exercised, and we have seen in the field of mathematical education major reform undertakings that were open to serious criticisms not on recondite grounds of educational or mathematical philosophy, but on the down-to-earth grounds of sheer mathematical and educational competence or conscientiousness.

Educational reform has this in common with medical research that it deals with the lives of human beings. Untold damage can be wrought if it deals with them carelessly and callously.

It is therefore appropriate to spell out explicitly what the exercise of proper care does mean in the field of mathematical education. To this I turn now.

First of all, there can be no excuse whatever for finding out from experiment with children what could have been found out by the mere exercise of careful judgment. Educational experimentation should not be a substitute for thought, nor a means for dispensing with the need for thought.

Thesis 11.

No major proposals should be implemented in actual practice —not even on an experimental scale—unless they have been elaborated with maximum care, and have had the benefit of careful and informed scrutiny and discussion. There can be no excuse for the use in actual teaching of hastily written, shoddy, obviously imperfect teaching materials.

Probably the worst example, considering the scale of the operation and the way in which it was advertised, is provided by some of the sample textbooks produced by the American School Mathematics Study Group—textbooks that were launched into large-scale use in actual teaching without even having had the benefit of responsible editing by responsible editors willing to sign their names.[1])

This example points up a further problem. The School Mathematics Study Group widely, and correctly, advertised the fact that respected professional mathematicians had been involved in its work. There have been several disturbing instances, in countries on both sides of the Atlantic Ocean, in which respected, and sometimes even very distinguished, mathematicians allowed themselves to be associated with, or even explicitly endorsed, novel textbooks which, quite obviously, they had not read carefully; textbooks which contained purely mathematical nonsense that any competent mathematician could detect.[2])

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1) See for instance my paper: Sampling a Mathematical Sample Text, Amer. Math. Monthly, Vol. 70, No. 4, April 1963, pp. 452-459.

2) G. Polya, Mathematical Discovery. Vol. II, p. 134, 14.16, makes the same remark in the form of a lovely anecdote.

In my view, this is exceedingly serious. The public at large—including, very often, the educational public—is nowhere more helpless and more dependent on reliable advice than when it comes to judging the mathematical correctness of proposed texts and programs. No non-mathematician, and very few practicing teachers, can challenge the word of a university mathematician who tells them that this or that proposal is "in the mainstream of contemporary mathematics". There arises therefore a heavy responsibility for the professional mathematician not, in effect, to endorse mathematical quackery. Indeed, he, and he alone, can expose it—he is at least responsible not to condone it.

Thesis 12.

It should be a matter of professional ethics for any mathematician not to endorse or lend his name to any mathematical textbook or method, unless he has satisfied himself of at least that text's or method's mathematical correctness and quality, or unless he explicitly specifies on the record whatever reservations he has to make.

Thesis 13.

New mathematical textbooks, particularly novel ones, should be carefully reviewed at least for mathematical content in the major mathematical journals.

I may add in passing that the amount of self-advertising allowed to the various reform undertakings might well be drastically cut down. It is inconsistent with well-established standards of editorial propriety when reputable journals, on the one hand do not review novel textbooks, yet on the other hand give the authors of these texts editorial space for advertising the alleged merits of their work. More generally, I believe that there are altogether too many publications that merely rehash the claims of the various reform undertakings uncritically, while there are far too few publications attempting to give a serious critical evaluation of them.

Once texts or curricular proposals are as sound as competent and knowledgeable thinking can make them, time has come to

try them out in the class-room. I say *try them out*! That is, even at that stage it would be utterly irresponsible to introduce them for large-scale use. The next stage must rather be one of careful small-scale experimentation, using the texts in a carefully selected sample of actual classrooms. The selection must be made in such a way as to accurately reflect the kinds of classroom situations in which the texts are to be used, if found to be successful; e.g. classes of gifted children or of average children; of highly motivated or of poorly motivated children; classrooms with highly qualified teachers or classrooms with poorly qualified teachers, etc. It is important that this experimentation be as little glamourized as possible; otherwise, the Hawthorne-effect will be maximized, and strong drives will be created, whether conscious or unconscious ones, to ignore or hide possible adverse results.

The use of a new approach only constitutes bona fide experimentation if there is careful evaluation of the resutls. This is elementary. Unfortunately, it is necessary to state this. There is a veritable mountain of publications offering, proposing, developing " new thinking in school mathematics " (to borrow the title of a well-known OECD publication) and proclaiming its miraculous therapeutic virtues. There are tragically, scandalously few publications giving a careful and candid account of observations made with the use of those new approaches. In fact, I can only think of one—a very interesting and informative paper by F. M. Hall, Group Theory in the Sixth Form, Math. Gazette XLV, 353, October 1961. This is probably not the only one; but it does belong to a pitifully small group. Can it possibly be that so few results are being reported because there are so few encouraging results to report ?

However that may be, I think that the publication of candid accounts of experiments is essential. Indeed, an " experiment " has only truly become that once its results are on the public record.

Thesis 14.

Careful, small-scale experimentation followed by candid, independent, competent evaluation in terms of aims stated beforehand

should precede any large-scale introduction of reforms. The results of these evaluations should be published in every case.

I may add two remarks here. First, the willingness to have one's results evaluated and the evaluation published really is the crucial test for the seriousness of a reform undertaking. No financial support should therefore be given to any organization or project that does not include *in its plans* satisfactory plans for the evaluation of results.

In many cases, particularly in the case of wholesale approaches to the teaching of mathematics, the evaluation might well include an appraisal by non-mathematicians involved in education, for instance concerning possible side-effects. Mathematics teaching is not the whole of education ! A proposal might for instance succeed on its own terms, but require an inordinate amount of time or home-work; or destroy the chances of establishing meaningful connections between mathematics and physics; or require groupings of children that are inherently undesirable, whatever their specific usefulness might be. These aspects should not be disregarded.

Contrarily to, say, history or literature, mathematics is a fairly international discipline, and the teaching of mathematics is a fairly international problem with solutions that vary relatively little from one country to another. If it is possible and makes sense to teach certain kinds of students in a certain way in one place, there is a strong likelihood that this will also be possible and make sense in other places, at least if these have the same basic aims of education. There is therefore a strong case to be made for an international pooling of resources and experience in the reform of mathematics teaching. By the same token, there is a strong case to be made *against* the proliferation of little local reform undertakings ignoring each other and everything that went on before them, duplicating each other, refraining from talking to each other, each one proceeding with similar lack of sensible evaluation. The reform of mathematics teaching must not become a form of international *featherbedding* for school and university teachers in need of prestige, summer earnings, and openings with textbook publishers.

Among the many dangers in that senseless, sometimes ridiculously nationalistic, proliferation, two are particularly conspicuous. On the one hand, genuinely valuable and significant work may simply be buried under the accumulation of worthless and monotonously similar work; or it may be shouted down in the contest for publicity by undertakings of less intrinsic merit, but greater financial strength and Madison-Avenue-type acumen. Where too much is going on, not as a coherent and focussed effort but as a disjointed collection of isolated ventures, it is very much as if nothing were going on: everybody talks and nobody listens.

In addition, when many ventures arise not because genuinely creative educators have an important contribution to make, but because there is a bandwagon which some local people would not like to miss, the result is the spread of a stifling and unreasoning orthodoxy, and beyond this, the development of an attitude of mind which is clearly incompatible with educational progress of any kind. I feel that this to some extent has been the fate of the movement for " modernization ". Whatever the intrinsic merits of the substantial proposals that have been made for teaching " modern " topics, there is no merit whatsoever in the spread of numerous " reforms " which are " modern " to the extent that they use the " language of sets ", and no more—indeed, occasionally use it in a way which makes clear that the authors of the reforms have no inkling of what it is that they are talking about.[1])

Thesis 15.

Appropriate steps should be taken for an international pooling of resources in the field of mathematical education: the pooling should extend over the whole range of the process of reform, from the first elaboration of new approaches to the final evaluation of their feasibility and merit. It should particularly also include the creation of new textbooks.

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1) A wide-spread general instance of this is the introduction of the " language of sets " by authors who are not aware of the basic difference between the *extensional* and the *intensional* idea of a set. These authors will then give a family, a class, a football team as examples of sets, unaware of the fact that from the intensional standpoint of ordinary speech these remain *the same* family, class, or team after the addition of a new member.

How can this be done in practice ? Not primarily, I believe, through organizational means. The impact of Parkinson's Law on the reform movement has been much too heavy as it is already. In fact, there is no need to devise new means. The means for establishing international cooperation in research are wellknown; they apply in the field of mathematical education as they do elsewhere. They all revolve around two foci: professorial chairs as centers of broad-based scholarship, research, and teaching; international journals as organs for the publication, review, confrontation, and discussion of work done.

Thesis 16.

To ensure continuous work in the field of mathematical education, at an appropriate level of intellectual distinction, the creation of a limited number of university chairs for mathematical education should be encouraged. The requirements for the holders of these chairs—both in terms of their qualifications, and in terms of the duties to be discharged by them—should be comparable in every way to the requirements for holders of chairs in other fields of learning. Necessary financial support for these chairs should include the provision of appropriate scholarships to enable students from all over the world to come and study under the holders of these chairs.

Thesis 17.

There is an urgent need for a genuinely international journal —international in its editing and in its diffusion—to deal with all aspects of the reform of the teaching of mathematics, at an appropriate level of editorial sophistication.

If we are to make enlightened choices concerning desirable ways of teaching mathematics, the first requirement is a wide range of thought and experience concerning conceivable alternatives. The development of a wide and diversified range of approaches of suitable quality should therefore be actively encouraged. It is shocking to find support being given to dozens of enterprises that all do essentially the same thing,

while dissenting approaches, far from being fostered, are being ignored and starved. It is even more shocking to find organizations making a loyalty oath to H. M. Nicholas Bourbaki in effect a prerequisite for support. Bribery is no way to foster the free growth of ideas, in education no more than anywhere else.

Thesis 18.

Every effort should be made actively to maximize genuine creative diversity in the field of mathematical education, and to maximize at the same time the intellectual confrontation between the various approaches.

The way to accomplish this is indicated by the theses 16 and 17. Ideally, the chairs envisioned in thesis 16 should be held by men or women who each represent one distinguished and distinctive approach to the teaching of mathematics. One could well envision, for instance, one such chair being held in Paris by one of the intellectual children of Professor Nicholas Bourbaki, as a center of thought and experimentation along lines that have become familiar in theory, if not in practice, to everyone concerned with the teaching of mathematics.

The confrontation will arise in three ways. First, if those chairs live up to the requirements of thesis 16, then confrontation of varying approaches, in seminars, study of the literature, guest lectures, etc. will be one of their primary concerns. It would be as unthinkable for a distinguished professor of mathematical education to confine himself, and the horizon of his students, to only the approach that he himself favours, as it would be for a professor of physics or biology. Second, the journal envisioned in thesis 17 would of course operate as a constant, challenging medium of confrontation and discussion. Finally, once enough insight has matured to warrant it, time has come for international congresses.

III. And what about " modernization " ?

I do not seem to have spoken about our topic at all—the so-called " modernization " of secondary school mathematics,