Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 11 (1965)

Heft: 4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: PRIORITIES AND RESPONSABILITIES IN THE REFORM OF MATHEMATICAL EDUCATION: AN ESSAY IN EDUCATIONAL METATHEORY

Autor: Wittenberg, Alexander

Kapitel: I. The Teaching of Mathematics

DOI: https://doi.org/10.5169/seals-39981

theses as I go along. A relatively elaborate attempt at detailed axiomatic analysis of the meta-problems will be found in a questionnaire in a forthcoming issue of DIALECTICA [1]).

I. The Teaching of Mathematics

1. The teaching. What a teacher actually does when teaching mathematics is determined by *two* parameters; I shall call them *content* and *form.*

(a) The *content* of his teaching is made up of the items included in the syllabus—e.g. equations, inequalities, functions, physical applications, etc. This also includes terminology (e.g. the " language of sets "), standard methods, etc.

(b) The *form* of the teaching can be analyzed into two different, although closely related, elements:

b 1) *Inner organization of the content.* The content can be presented in a coherent or an incoherent way, with an atomistic or a holistic structure, as a collection of many isolated items (" units ") or as an exploration of a few major connected topics (in the sense of the " Themenkreis-methode " which I discuss in my book " Bildung und Mathematik ".[2]))

b 2) *Type of classroom work.* The major emphasis in the actual teaching process can lie on the communication of ready-made mathematical information by the teacher to the student (dogmatic, magisterial teaching); in that case, the underlying structure of the teaching situation is: active output by the teacher, passive reception by the student. (The student may still by very *busy* in such a situation —but his busyness is embedded into a basic passivity; it consists in busily carrying out what he has been told to do, in the way which has been prescribed to him). The role of the teacher essentially is that of a mechanical purveyor of information and checker of routine problems. Not surprisingly, many will think that this operation can

---

1) Dialectica, forthcoming.
2) Ernst Klett Verlag, Stuttgart, 1963.

be mechanized altogether, and the teacher's job done more cheaply and more efficiently by a teaching machine.

Alternatively, the major emphasis can lie on the generation by the teacher of broadly directed, yet largely autonomous intellectual activity on the part of the students: genetic teaching, rediscovery method, méthodes actives, Arbeitsunterricht.[1]) The students think by themselves, and are ultimately guided by their own personal insight. The primary function of the teacher in this case is to question, to guide, to stimulate, to catalyze, to help synthesize and to assess within a broad predetermined framework of teaching aims: The teacher makes the students think creatively. In this case the details of the teaching process cannot be completely predetermined; they are to some extent essentially unpredictable. A fortiori it is impossible to *program* this kind of teaching. Between teacher and students, a true interpersonal relationship develops as a support for, and integral part of, this educational process—a genuine, continuing dialogue, whose educational value extends far beyond the mere acquisition of mathematical knowledge by the students.

Thesis 1.

In the teaching of mathematics, the form *of the teaching is more important than the* content. *In other words:* how *the student learns is more important than* what *he learns.*

Thesis 2.

The teaching of mathematics should maximize the inner organization of the content; it should present the content as a meaningful, coherent, interconnected exploration of a few clearly significant topics. At the same time, the teaching should maximize autonomous intellectual activity by the student—insight, understanding,

---

1) See G. Polya, How to Solve It, Anchor paperback, and Mathematical Discovery, Vol. I and II, Wiley. M. Wagenschein, Exemplarisches Lehren im Mathematikunterricht, Der Mathematikunterricht 8, 1962, part 4. A. Wittenberg, *op. cit.*, and A. Wittenberg, Sr. Ste Jeanne de France, Fernand Lemay, Redécouvrir les Mathématiques, Neuchâtel, 1963. Further literature is cited in these books.

creative problem-solving, active intellectual analysis and synthesis of genuinely interesting mathematical situations.

Thesis 3.

Reform of the form *of teaching mathematics should take clear precedence over reform of the* content. *The latter should be subordinated to the former; that is, a minimum requirement for proposed changes in content is that they should be consistent with, and a fortiori should not jeopardize, desirable changes in form.*

To exemplify the impact of the three theses in a nutshell, they mean that it is more desirable, for the mathematical *education* of a student, to acquire a thorough, versatile, resourceful understanding of elementary algebra, and no more, than it is to acquire insightless skill in the performance of integrations and differentiations.

The three theses also imply that reforms that consist in a mere increase, or change, in content can be highly misleading, creating a façade of improvement over what may actually be a deterioration of the teaching. This will occur, in particular, if the changes in content force a change in form away from what is desirable.—One of the most typical examples for this is the set-theoretical approach to geometry, in which the student is required to take for granted that a line, or a plane, is a set of infinitely many points; an assertion that raises a number of very old puzzles, and one that a student can only be made to accept unhesitatingly if he is in effect prevented from thinking by himself.[1])

As to the content of the teaching, an essential minimum requirement is that it should not be the result of an arbitrary choice. The motivations for a given choice of content can be of several kinds—intrinsic interest, practical necessity, professional preparation of the student, etc. Whatever they are in any given case, they should be bona fide motivations, they should be explicitly spelt out, and the connection between these motivations and the actual curricular proposals should be explicitly

---

1) See A. Wittenberg, Formation et Information Scientifiques dans l'Enseignement secondaire, Revue Internationale de Pédagogie, Vol. 9, No. 4, 1963-64, pp. 407-417.

analyzed.[1]) It goes without saying that the analysis must also extend to the compatibility of the proposals with thesis 3:

Thesis 4.

The content of the teaching must be an effectively stated function of effectively stated motivations; the implications of any given content for both aspects of the form of the teaching must be explicitly analyzed.

2. THE TEACHERS. No teacher can teach in the way required by thesis 2 without thorough personal mastery of the subject-matter taught. Without such mastery, he will be at the mercy of every unexpected remark by a student, of every original suggestion, of every surprising question. Lack of grasp of the subject-matter compels the teacher to minimize personal and original thinking on the part of the students; this becomes for him a matter of sheer self-preservation. In addition, it is the most basic fact of life in education that no teacher can convey an understanding of, nor a taste for, that which he does not understand himself.

Thesis 5.

No teacher should teach mathematical subject-matter unless he has thorough personal understanding of that subject-matter in its broader context.

For instance, no teacher should teach an axiomatic or quasi-axiomatic approach to geometry, who has no clear insight, both into the meaning of such an approach, and into the nature of the axiomatic method. A teacher should only teach sets if he knows " naive set theory " reasonably well, and has personal knowledge of at least some significant examples of important applications of the set-theoretical approach; etc.

An immediate consequence of thesis 5 is the following:

Thesis 6.

The requirement of thesis 5 should be the primary, overriding factor governing the choice of content. In particular, introduction

---

[1]) Cf. the Dialectica-questionnaire, footnote 2.

of new content should be subordinated to the availability of teachers satisfying the requirements of thesis 5.

This means, for instance, that if in a given situation a choice must be made between having traditional elementary algebra taught by teachers who have or can acquire a thorough understanding of it, or having some brand of " modern " algebra taught by teachers without such understanding, the decision *must* be in favour of the former possibility, irrespective of any other elements influencing the choice.

Rethinking the teaching of mathematics in the sense of thesis 2 is a difficult and sophisticated intellectual exercise; so is the mastery of new and unfamiliar mathematical material. To do both things at once is nearly impossible for many teachers. In the light of the priority of form over content, that fact entails the following:

Thesis 7.

When educating or re-educating teachers, priority should be given to the rethinking of the teaching of familiar material.

Some critics of my book " Bildung und Mathematik " have overlooked the fact that it is this thesis which, quite explicitly, underlies the choice of the material discussed in the book (see pp. 70-71).

3. The results. No rational discussion of the teaching of mathematics (or, for that matter, of the teaching of any other discipline) is possible without clear criteria of success. If we do not know what it is that we are trying to do, we cannot evaluate whether we succeed in doing it or not, nor even whether we are *in fact* attempting to do what we *believe* we are attempting to do. For instance, we may believe that we are trying to teach an understanding for the mathematical idea of structure; while in fact, conceivably we are not even making a beginning in that direction. In the teaching of mathematics as elsewhere, wishful thinking and self-delusion are no substitute for careful, critical, in effect axiomatic, analysis.

Thesis 8.

Any proposal for the teaching of mathematics, and in particular any reform undertaking, must state beforehand *the terms in which it defines its success, and the means by which it proposes to check whether it succeeds or not. Its proposed criteria must be demonstrably consistent with, and a check upon, its stated aims.*

The most straightforward (although not necessarily the only) check on the actual aims of our teaching is provided by the actual content of our examinations. Essentially, we are really teaching that which we are prepared to examine in our students. It is a sorry and a ridiculous sight when reform undertakings define themselves in the most ambitious and uninhibited terms, claiming to teach everything from the axiomatic method to the structural approach to mathematics, and then end up setting final examinations in which the students are required to do nothing more subtle than, say, find the cardinality of the union of two sets of given cardinalities.

Thesis 9.

Examinations must be framed in terms of the stated aims of the teaching. The first test of the seriousness of the commitment of those responsible for the teaching to their stated aims lies in their willingness to devise examinations that are explicitly geared to these aims.

But candid and searching examinations are of no avail if the examination *results* are manipulated in such a way as to effectively destroy the value of the examination as a control on the success of the teaching. In particular, it is a rather crude statistical fallacy (and one that is particularly surprising when it comes from leaders in the movement to introduce the teaching of statistics in the schools !) to adapt the results of examinations on new curricula to standard statistical norms, and still expect to get from these examinations useful information concerning the value of the curricula.[1])

[1]) See Cooperative Mathematics Texts, A Progress Report; Mathematical Education Notes, Amer. Math. Monthly, 69, 3, March 1962.

Thesis 10. (Corollary to theses 8 and 9).

Proposals for the teaching of mathematics must include proposals for typical examinations, together with a statement of the kinds of examination results that will be considered acceptable *in terms of the success or failure of the proposals.*

I may add here that, if the aims include the mastery of *ideas* by the student (for instance an understanding for the idea of mathematical structure), then the examinations must include tests for mastery of ideas, that is essay-type questions; e.g. " Explain and discuss the idea of mathematical structure ". In a system of education like the French one, such questions might build valuable bridges between the teaching of mathematics and that of philosophy, particularly philosophy of science.

II. The Process of Reform

So far, I have been discussing basic principles that, in my opinion, ought to govern the process of elaborating, implementing, and assessing proposals for shaping the teaching of mathematics.

The most basic principle of all, however, has not been stated so far. This simply is that *proper care* must be exercised in shaping the teaching of mathematics. Everything else will be to no avail if such care is absent, if we allow sloppiness and irresponsibility to prevail; exactly as progress in medical science will be of no avail if medical practitioners fail to wash their hands and to learn about proper dosages of antibiotics. In the past, unfortunately, proper care has not always been exercised, and we have seen in the field of mathematical education major reform undertakings that were open to serious criticisms not on recondite grounds of educational or mathematical philosophy, but on the down-to-earth grounds of sheer mathematical and educational competence or conscientiousness.

Educational reform has this in common with medical research that it deals with the lives of human beings. Untold damage can be wrought if it deals with them carelessly and callously.