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II. Let the rank of Q be k, and centers exist. Then these centers are solutions of

$$\xi_k = \xi E = -\left(\frac{\partial f}{\partial \xi}\right) E Q^{-1}, \tag{3.2}$$

where Q^{-1} is the reciprocal of Q, see [2]. That is, if E is the projection on the range of Q, then

$$Q^{-1} Q = Q Q^{-1} = E .$$

Here we choose the center of quadric curvatures at a point of (3.2) so that, it is at the shortest distance from γ.

III. When the rank of Q is k and the quadric does not have centers, then we say that f does not have a center of quadric curvature.

4. Direction of quadric curvature

In part I and II of section 3 we respectively call the vectors ξ and ξ_k the directions of quadric curvature of f at $(c_1, \ldots, c_n)$. In III of section 3, we define the direction of quadric curvature to be a vector δ which satisfies

$$\delta = \delta E = -\left(\frac{\partial f}{\partial \xi}\right) E Q^{-1},$$

where E is the projection described in section 3.

5. Vertex points

Let at the point $\gamma = (c_1, \ldots, c_n)$ of f the direction of quadric curvature be the same as the normal to $f = 0$. Then γ is called a vertex point of the function f.

Theorem: A necessary and sufficient condition for a point to be a vertex point of the function f is that at that point

$$PQ = QP,$$

where P and Q are the matrices described in section 3.