5. FURTHER OBSERVATIONS

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many cases differ from those in abstract mathematics. For instance, in statistics it is difficult to acquire operational skill as long as one has not really and independently understood the fundamental notions. "

5. Further observations

Perhaps the major motivation for teaching modern mathematics, or mathematics in a modern spirit in high school, is to prepare the student for his university experience. The need for this is particularly well brought out in a quotation from the French report from Professor Lichnerowicz. The quotation (in translation) reads: " The classical teaching of our lycées in a large measure conditions our students to a certain conception of mathematics, a conception which is ... derived from the Greeks, and ... from the experience of mathematicians of the middle of the nineteenth century ... At the university, the students suddenly encounter the spirit of contemporary mathematics, a painful shock ... The student must totally ' recondition ' himself ... and this is translated by an expression which I personally have often heard: ' What you are teaching is no longer mathematics ' ... "

I am sure that many of us can testify to the same experience. Let us now examine a few pedagogical problems.

The Netherlands report recommends that " stress should be laid on thinking mathematically and more value attached to this ability than to knowledge of a variety of less important facts. " If this philosophy is adopted, then presumably the exact choice of topics is not nearly as significant as the manner in which they are presented in the high school.

An important pedagogical idea is expressed in the Portugese report: " For this introduction (of modern mathematics) it would be essential to bring out many concrete examples, well known and quite suggestive, as well as amusing, and one would be careful not to introduce formalism until one was sure that the student had grasped the ideas behind them. "

One question that arises in the introduction of new topics is what topics are reduced to make room for the inclusion of new ideas. By far the most frequently mentioned topics were a reduction in the amount of time spent on synthetic geometry, a considerable reduction of trigonometry, especially the emphasis on triangle solving, a reduction of solid geometry, possibly by incorporating it into the first course in geometry, and a reduction in some of the traditional and not very practical numerical methods included in algebra courses.

A pedagogical question on which there seems to be considerable disagreement is the extent to which high school mathematics should be axiomatized. I found several recommendations that there should be a substantial extension of the body of axiomatics in high school, or even that axiomatic systems, as such, should be studied. On the other hand, there were about an equal number of objections to excessive use of axiomatization in the modernized curricula. For example, " The enrichment of the syllabus by the insertion of interesting examples of modern elements of mathematics is to be encouraged, and indeed is bound to happen. But the systematising of teaching in line with axiomatic mathematical theories would lead to a situation contrary to accepted British teaching principles. "

A different view concerning axiomatics is shown in the French report: " *Axiomatic Exposition.* A program cannot demand that teaching have an axiomatic character until sufficient scientific experience permits the student to feel its need. Axiomatic procedure is extremely rigid, each step is strictly controlled, appeal to the intuition has no value because the choice of axioms accepts some facts and rejects others just as sympathetic to our intuition. If the construction succeeds and gives what our experience of the question expected, if one has more or less demonstrated the independence of the axioms and the categorical quality of their set, one sees that the choice was good. But who will believe that such a choice can be made without fumbling ? and different axiomatizations are valid. It is impossible to set them forth without dogmatism, without appealing to the authority of the teacher who is able to show only to the end that the work is valid.

" In secondary school teaching one can only try to come to the conclusion that axiomatics are doubtless possible and desirable in mathematics. In terminal classes, it is recommended to do a few axiomatic expositions at the outset, granting the necessity of afterward accepting a more technical viewpoint. But it is very dangerous to do partial axiomatics, which hide the unity of mathematics even if one doesn't make vicious circles (like using number to axiomatize geometry and geometry to axiomatize the notion of number !).

" However, even if a large place is left to the intuition of the children and the path chosen for exploring the program is flexible and takes account of the spontaneity of the students . . . it is necessary for the teacher to impose an order without which there would be only confusion. This order reflects an underlying axiomatization adopted by the teacher, of which the best pupils can become aware at the end of the school year. "

In the historical development of mathematics, it is usually, though by no means always, the case that a certain body of mathematical facts is first discovered, and then one or more people perform the very important task of systematizing this information by specifying a minimal number of axioms and deriving the other facts from these. It is therefore clear both that some acquaintance with axiomatic mathematical systems is an important part of mathematical education, but also that mathematics is something over and above mere development of axioms. Just what the happy compromise is between these two trends may be a topic well worth discussing at the Congress.

The newly developed Danish curriculum provides a very interesting idea — namely, an optional topic to be selected by the high school teacher. The choice of this topic is described as follows:

" Contents, extent and mode of treating the optional subject should be adapted in such a way that the students are not in this field faced with more difficult problems than those arising from the other lessons of mathematics.

" Some examples of the fields from which the optional subjects may be taken: History of mathematics, number theory, matrices and determinants, theory of groups, set theory, Boolean algebra, differential equations, series, probability theory, statistics, theory of games, topology, projective geometry, theory of conics, noneuclidian geometry, geometry of higher dimensions, geometrical constructions, descriptive geometry.

" The optional subject may also be chosen in connection with the corresponding part of the physics course. As examples of suitable subjects may be mentioned: Probability theory and kinetic theory of gases, differential equations and oscillatory circuits. Finally the optional subject may be organized in connection with other subjects than physics, e.g., probability theory and heredity.

" The program for the optional subject will have to be submotted to the inspector of schools for approval.

" The existence of an optional subject in the mathematics curriculum is new in Denmark. This subject will have such an extent that a couple of months in grades 11 or 12 will be occupied by it. Of course, both modern and classical subjects will be chosen, but it is expected that many teachers will choose the theory of probability as their teaching subject. In the list of non-optional subjects probability does occur, but only on a very modest scale. Of course the teacher is free to choose between an axiomatic and a non-axiomatic treatment of probability, but certainly an axiomatic treatment will be used by some teachers. (This will probably be easier to carry through if one restricts oneself to discrete sample spaces.) In this case the pupils will get a very useful impression of a simple axiom system and an example of a mathematical model. "

One topic mentioned in a number of reports is the extent to which calculus is included in the secondary school curriculum. I have not specifically discussed this topic since it cannot legitimately come under the heading of " modern mathematics ". However, it is clear that there are increasing pressures from physical scientists to teach some units in calculus in our secondary school curricula, and to a great extent this pressure may compete with the demands for modernizing of modern mathematics. Let me simply indicate that at the present time there are vast differences from the majority of countries that teach no calculus at all in the secondary school to the large number of countries that teach a first, more or less intuitive introduction to calculus, to such extreme examples as the recent experiment in Sweden. A special experimental unit will be taught in that country on differential equations: " This small course consists of linear equations of first order and of second order, with constant coefficients. Proofs of existence and uniqueness are given. "

The Hungarian report calls attention to two problems that have caused difficulties in modernizing the high school curriculum: " One is the preparation of the teachers now teaching for the handling of new subjects. Without this, the introduction of such topics cannot succeed. But equally important is the formation of public sentiment, since for the majority of people it is not obvious why their children in high school should learn about problems that their parents may never have heard of in their entire lives. We have to solve these problems simultaneously with the modernization of the curriculum. "

I am quite certain that many reporters would heartily support these remarks. There are indications in many reports that major national attempts have been made to modernize the training of existing high school teachers. This is, of course, often a highly painful and difficult experience for adults who have left their universities with the impression that they are prepared to teach mathematics for the rest of their lives, and find themselves forced to return to study what often seems to them strange new ideas.

Speaking for the United States, I may add that the problem of informing parents of high school children is equally critical. In many communities where the schools were happy to modernize the mathematics curricula they ran into unexpected opposition from parents who simply could not understand why modern mathematics should be taught, or even how there could possibly be such a thing as modern mathematics. It is strange that, in an age of fantastically rapid development in mathematical research, perhaps a majority of laymen are under the impression that all new mathematics was done hundreds of years ago.

Most of the reports were from countries with educational systems based on centuries of tradition. I was fortunate in obtaining one report from Africa, which painted a fascinating picture of the problems faced by newly developing nations. I would like to reproduce just one quotation which I found particularly interesting, from the report of Sierra Leone:

" The most important factor in our survey is that in all these areas education has been expanding very, very rapidly within the last ten years. The number of secondary schools has at least doubled in all areas and is still expanding. It is in these new schools that there is the greatest opportunity for introducing modern mathematics. The teachers in these schools are usually young enthusiasts and, the schools often being in new towns, are sufficiently separated from the older traditional schools to make it possible for experimental work to be carried out without pupils and parents continually comparing the work there with the work being done in other schools. "

6. Conclusions

It is clear from the reports that many nations have made an excellent start on the modernization of high school mathematics curricula. It is equally clear that much hard work still needs to be done.

There seems to be fairly general agreement that some basic concepts from set theory and logic should be introduced, that geometry should be modernized, that some elements of modern algebra be introduced, and that probability and statistics are suitable for high school teaching. Even more important is the general agreement that much of traditional mathematics should be taught from a modern point of view. However, as far as the details of these recommendations are concerned, there is considerable disagreement.

The two greatest difficulties blocking progress are the critical shortage of qualified teachers, and the lack of suitable text materials. The former problem has been attacked in a few countries by running special courses for high school teachers whose training was mostly traditional. The latter is being solved by the writing of many excellent experimental text materials.

I should like to conclude the report by making two specific recommendations to ICMI: