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2. The process of change

Only a very few countries reported that so far little or no attempt to introduce modern mathematics had taken place. Of course, this small number may not be significant, since my sample is biased: Presumably countries in which absolutely no attempt to modernize mathematics has occurred have not filed reports on this topic.

Of the remaining countries, the vast majority report that the attempts to modernize the curricula have consisted mostly of informal discussions amongst mathematics teachers and a number of highly encouraging experiments by individual teachers. It seems to be a universal experience that attempts to teach selected topics from modern mathematics well, in reasonable quantities, can be highly successful.

I shall discuss in somewhat more detail reports of a few countries where national reform movements have taken place.

France had a head-start over most other countries in that the French secondary school mathematics program was even traditionally unusually strong. The typical secondary school teacher in France had a strong university degree in mathematics which both placed France into a good starting position and made it easier to introduce modern ideas. Reform started with a series of experiments by teachers trying out various topics of modern mathematics in the classroom. This led to the writing of a series of articles and monographs which were widely discussed. Eventually, a number of seminars were formed at which secondary school teachers and college professors together discussed pedagogical problems involved in curriculum reform. France is fortunate enough to have persuaded a number of its very famous mathematicians to give lectures to high school

teachers on topics of modern mathematics. This is all the more remarkable, in that all of this work, both on the part of the lecturers and the high school participants, was entirely voluntary, without compensation. All of this effort finally resulted in success: The Ministry of Education gave its official blessing to plans formulated for the modernization of the secondary school curriculum.

There is also a report of some experiments in France with children of a younger age, to present some basic ideas of geometry, number, and sets from a modern point of view.

Curriculum reform in Germany is complicated by two factors. First of all, in the Federal Republic the problem of education is not in the hands of the Federal Government but of the individual States. Therefore, it is very difficult to initiate a national reform. A permanent conference of ministers of education has been established to provide some degree of uniformity in school curricula. A second complicating factor is the existence of three types of gymnasiums in Germany, with quite different attitudes towards the teaching of mathematics. Real reform has been possible primarily in the mathematics-science version of the gymnasium.

On the other hand, the German gymnasium covers a nine-year period and therefore can provide a continuity in mathematical instruction not possible in most other countries. The German report points out a problem common to many nations —that the amount of time allocated to mathematics in the curriculum is severely limited. Therefore, the introduction of modern mathematics cannot be thought of as the addition of new topics to an existing curriculum. Rather, one must find topics within the traditional curriculum which, although they may have been worthwhile, are not from a modern point of view indispensable. Modern ideas are introduced by the replacement of such topics with selected ideas from modern mathematics. On the other hand, one often has an opportunity to supplement these topics for the better students in an " Arbeitsgemeinschaft ", where students voluntarily go deeper into the subject matter. Apparently such informal courses play an important role in the education of mathematics students in Germany. Not only does

Germany propose a new curriculum for high school mathematics, but their report shows evidence of deep thinking on individual topics in this curriculum. A number of extremely useful articles and monographs have been written in Germany, and the reader will find in the appendix of this report a bibliography from the German report.

The status of Italy seems typical of a large number of countries. Two national commissions have studied the problem of modernizing the high school curriculum, and have reported their findings. Italy is now ready to start implementing these recommendations.

In Israel the Ministry of Education has appropriated funds for the writing of experimental textbooks by a group of mathematicians at Hebrew University.

Poland is an example where, although relatively little actual experimentation has been done in the classroom, there has apparently been an immense amount of highly constructive discussion amongst the teachers of mathematics. The Polish report gives every evidence of having had topics discussed both in a wide range and in great depth; and of highly laudable, constructive thought on the part of many mathematicians. The report indicates that these plans have now reached the stage where they hope to try out experiments on a variety of different lines in the classroom.

A most interesting cooperative enterprise is under way in the Scandinavian countries. They have formed a " Scandinavian Committee for the Modernizing of School Mathematics ". This represents a cooperative effort amongst Denmark, Finland, Norway and Sweden to pool their resources, both mathematical and financial, for the improvement of mathematical education. This is made possible not only by the geographic proximity of these countries but by strong similarities amongst their educational systems, as well as traditional ties.

In 1960 the Committee adopted a 5-point program: (1) To survey mathematical needs both for the use of industries and for the needs of universities. (2) The development of new mathematical curricula. (3) The writing of experimental textbooks. So far four monographs have been produced. (4) Plans

have been made for extensive testing of these experimental materials. (5) After these tests have been concluded, the Committee is to make official recommendations to the four governments for the adoption of new curricula for secondary education.

The United States has been unusually fortunate in planning its development of modern mathematics curricula. Reforms of early university mathematics education were being planned a decade ago in the United States. These created new demands for the modernization of high school curricula. A Commission on Mathematics was established and worked through the mid-1950s under the chairmanship of Professor A. W. Tucker, Princeton University. While this Commission had no official national standing, its report has been widely read and has been immensely influential. [Copies may be obtained from the Educational Testing Service, Princeton, N.J.]

As soon as this report was published, it became clear that at least two steps had to be taken to make any reform in the United States a reality. One was the introduction of suitable text materials, even if they were of an experimental nature. The second was the training of tens of thousands of high school mathematics teachers who had never been exposed to modern mathematics. Here the National Science Foundation came to the aid of the mathematicians. Through grants, amounting to many millions of dollars, the National Science Foundation established means of meeting both of these problems.

First of all, special institutes were established for the retraining of high school mathematics teachers. Each summer thousands of mathematics teachers are enabled to study modern topics in mathematics with all their expenses paid by the Foundation. More recently, the Foundation has enabled mathematics teachers to return to universities for an additional year's study.

The writing of experimental text materials was started by various university groups, notably one at the University of Illinois. More recently, the National Science Foundation made possible the setting up of a national writing group, the School Mathematics Study Group, under the leadership of Professor E. G. Begle, originally of Yale University and now of Stanford University. Over a period of five years more than 100 mathe-

maticians and mathematics teachers have cooperated in the writing of a series of experimental materials. These have been widely tested throughout the United States and have been rewritten until they form both highly acceptable experimental text materials and will form a basis for future textbooks on the subject. (Information about these materials can be obtained from the School Mathematics Study Group, Stanford University, Stanford, California.)

The problem of implementation is made infinitely more complex in the United States than even that noted in the German report, since the final decision on curricula in most cases is neither in the Federal government's hands, nor in the hands of State governments. The latter usually set minimum standards, but the details of curricula are voted on by each individual community. Therefore, before reform is complete, many thousands of local school boards have to be persuaded of the desirability of modernizing their mathematics curricula. On the other hand, this local control also had its advantages in starting wide-scale experimentation. In many states it would have been impossible to get the State governments to approve the new curricula, because of lack of qualified teachers, but individual cities or towns were able to adopt new topics without waiting for State approval. We therefore find a strange situation in the United States, where one may find hundreds of schools with perhaps the most modern mathematics curricula in the world, and at the same time still find thousands of schools that have not even given any thought to the modernization of high school mathematics teaching.

In conclusion, I would like to reiterate a sentiment contained in the German report, namely that it takes at least a generation to complete a major change in the mathematics curriculum. At the rate mathematics is developing, by the time the present reform is completed, we are sure to want a reform of the " modern curriculum ".

This is perhaps dramatically illustrated in the United States by some exciting experiments carried out in the last three or four years in teaching modern ideas to students in the first six years of school. For example, in the city of Cleveland, a number

of suburban school systems adopted School Mathematics Study Group materials, starting with the 7th year of school, and have developed their own materials for the first six years. They are now facing the very serious problem that by the time their students have studied modern mathematics (in an elementary version) for the first six years, they will find the " frightening " new ideas of the 7th and 8th years much too easy, and hence these schools will find the modernized curricula terribly old-fashioned.

3. The new curricula

The most striking feature of the 21 reports is the degree of similarity in the proposals for including new topics of mathematics.

There are four areas of modern mathematics that are recommended by a majority of the reports. These are elementary set theory, an introduction to logic, some topics from modern algebra, and an introduction to probability and statistics. Equally frequent is a mention of the necessity for modernizing the language and conceptual structure of high school mathematics.

Perhaps the most frequently mentioned topic is that of elementary set theory. The concept of a set, as well as the operation of forming unions, intersections, and complements, constitute a common conceptual foundation for all of modern mathematics. It is therefore not surprising that almost all nations favoring any modernization of the high school curriculum have advocated an early introduction to these simple, basic ideas. An attractive feature of this topic is that in a relatively short time a student may be given a feeling of the spirit of modern mathematics without involving him in undue abstraction.

It should, however, be noted that in most cases only an elementary introduction of this topic is recommended. For example, the usual " next " topic in developing set theory is that of cardinality. Only three nations have suggested this as a possible topic for inclusion in the secondary curriculum.