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# EDUCATION OF THE TEACHERS FOR THE VARIOUS LEVELS OF MATHEMATICAL INSTRUCTION[1]

By Kay PIENE, Oslo

Some time ago, I met an old friend, a mathematics teacher. Years ago we were students together at the university. He is now a very capable teacher. He said: " I see you advocate a new program in mathematics instruction in schools ". " Yes," I said. " But if this program is introduced in our schools, I can not teach mathematics any more."

I think this sad story tells us how important the *training* of our mathematics teachers is.

When I was given the honored task of preparing a report on training of mathematics teachers at this conference, I realized that I had to base my comments on information from the different countries represented in I.C.M.I. On the other hand, I also realized that information on *how* teachers are trained *today* is not what we want. We are certainly more interested in knowledge of how—according to experts in these countries—this training *ought* to be organized.

I sent a questionnaire to the different national sub-commissions of I.C.M.I. and got answers from around a dozen countries. Not much, but many of them were long and thorough and have given me many ideas, even if the " ideal, future side " in the answers could have been stressed more. Besides I have got valuable information from other sources. By chance this summer Stockholm has had international teacher congresses arranged by W.C.O.T.P. and F.I.P.E.S.O. (World Confederation of Organizations of the Teaching Profession and Federation Internationale des Professeurs de l'Enseignement Secondaire Officiel). One of the themes discussed here was The Training of Secondary

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1) Report presented at the International Commission Mathematical Institution, Stockholm, August 1962.

Teachers. I have read and used the reports from the different countries, the summary report and the recommendations from the congress.

What I am going to say now is not the union, or not the intersection of all elements in the answer-sets. I alone am responsible, even if I am, of course, influenced by the answers.

First some general remarks:

In some countries—Sweden is one—the word mathematics is a word covering arithmetic and mathematics in the older meaning of the word. $7+8$ or $7 \cdot 8$ are here parts of mathematics. I prefer to use both words: *mathematics* when we have proofs and letters besides numbers; *arithmetic* when we have numbers alone and rules for those operations we define for them. (I am tempted in the old saying " Die Franzosen sagen pain, aber es ist doch Brot " to substitute for the placeholders: Swedes, mathematics and arithmetic).

A mathematical teacher is far from a unequivocal concept. We must know the type of school where he is teaching.

In most countries we find a school system with different levels. First, a primary or elementary school level with arithmetic, but where mathematics can be started. Looking back we find this tendency: mathematics is moving down and down. What 100 years ago was taught at universities, we now find in secondary schools.

In accordance with these principles we now find in many countries (real) mathematics starting already in primary schools.

All future primary school teachers today get some instruction in mathematics (and arithmetic) during their training, but this situation will put new demands on them.

In the first years of the primary school, we must have teachers who teach *all* subjects, but later on we must specialize. Not every primary teacher is able to teach mathematics, even on this level. I therefore think Denmark has found a good solution, having an *elective* subject in its training program for primary school teachers. Those teachers who like mathematics can take it. They have also developed a good plan for mathematics in 3 parts: 1) Parts of the high (secondary) school curriculum, 2) " professional insight and deeper understanding " (logic, set

theory etc.), 3) deeper treatment of some chapters from the primary school program.

But I am not going to discuss the training of primary school teachers any further. We return to our model of the school system in different countries.

The secondary level mostly first has a more general part, then a more specialized (gymnasium, lycée, college etc.) leading to college or university, at the same time giving the highest general education.

The upper secondary school again in most countries is divided into branches—humanistic, classical, modern, natural science, mathematical, commercial, technical, etc., with different demands in mathematics.

To simplify, I assume this model of a secondary school:

1) a first undivided level (11 to 12-15 years)
2) a second level (15-18 to 19 years)
   a) with a branch specializing in mathematics
   b) and other branches having mathematics as a less important subject.

It should be quite clear that these three school levels must or can have different mathematics teachers.

Our final aim is to give all countries "*good*" *mathematics teachers*. I do not think it is easy to define a "good" teacher—you cannot find a definition accepted by everybody. On the other hand, it is not an undefined concept. We have at least some ideas of what we mean, we know that some teachers are better than others. It is therefore also impossible to give *sufficient* conditions for the training and education of a good teacher. But we can give some, more or less *necessary*, conditions, which I divide into four groups.

1. The mathematics teacher must have a certain *general education*. We cannot have teachers with good knowledge of mathematics but who are outside that field rather ignorant. He should know one or two foreign languages, history, social science, politics, at least one arts subject and be prepared to share the interests of his pupils outside mathematics whatever they are.

It should be possible to give this general education at (*high*) *school* but at the university the student must have an open mind and be willing to widen his area of knowledge in these and other fields, but I would not have special courses in general education at the university.

Only one exception: If these courses are not given in the secondary school, I would in the first university year have a course in philosophy, especially in theory of knowledge, and one in general psychology which could also give insight into methods of learning and studying at a university.

2. The next two groups should cover *what* to teach and *how* to teach this material. We may also use the names: *academic* and *pedagogical* training. Both are necessary. First it is absolutely necessary that the mathematics teacher should know not only the material that he presents to his own students, but that besides he should have knowledge going further and deeper.

A well known Swede, representing for years mathematics in the Board of Education, said the other day, when he retired from the Board: " A teacher must not only be the best in his class, he must be sovereign in the knowledge material in the textbook ". He must be able to answer questions from his students which are more advanced, but he must also know the foundations of his subjects, the structures; he must know the working methods of mathematics, have a sure knowledge of such elements as definitions, postulates, theorems, problems, and of deduction and proofs, etc.

We understand therefore why the academic courses must have a very large part of the training, especially since mathematics has a solid position in the schools in mostly all countries.

First, it is quite clear that the amount of " what " will change from school level to school level.

Another question is not so clear. In most countries the academic courses are given at universities, in others we find special institutions for future teachers (Poland).

Relatively fewer and fewer of the university students of mathematics in these days become teachers. The teachers may have some specific needs. It is therefore understandable that

some countries have formed special training institutions for future teachers. But I think people going into research or industry more or less have the same needs as future teachers, they also need pure *mathematics*, and besides, I do not find it wise to force the students when they are as young as 18 to make a choice between school teaching and another mathematical career.

It is very important that the academic courses are given to students by mathematicians who have done research work and are real scientists, and if possible also have been school teachers and know the problems of the classroom. This was often the case in the old days when many university professors (like Weierstrass) started their career in a secondary school.

One solution would be to have a scientific representation of the subject matter supplemented by practical comments by an experienced and capable school teacher showing the best way of using the material in a classroom (as an example, introduction of positive and negative numbers or of complex numbers).

Some courses may still be left being in between the academic and the pedagogical courses. Italy has courses based upon Klein's *Elementarmathematik vom höheren Standpunkt aus* and Enriques' *l'Enciclopedia delle matematiche elementari.* In other countries you find so-called foundations courses, courses giving the background for the curriculum in schools. Such courses could best be given by school teachers who at the same time are mathematicians, but here every country must find its own solution.

In some countries there is a sort of an atomic system for the academic courses with many smaller courses, each ending with an examination (mostly a written one). In other countries we also find smaller courses, but only one, two or three examinations covering a group of such courses.

I think it is irrelevant for our main problems to discuss which of these two systems is best. As a model we may consider a university organization where it is possible to take mathematics courses on different levels. The first level A would correspond to level I for schools; the second level B for level II, and the third level C for level III. I assume that courses on

level A and B have examinations which are evaluated, and that to level C there will be attached some longer work which demands *days* (*or months*), that is, to write a sort of thesis or solve a larger problem or " show the relationship between the results obtained by several authors " (Dutch report).

To get a degree it is necessary in all countries to have taken a certain number of courses and passed the corresponding examinations. These courses can cover mathematics alone or one or more courses in other subjects.

A study of mathematics and nothing else would be too one-sided. For level C I would suggest one more subject, one which applies mathematics (physics, biology, sociology, psychology), but in this case I like to give the students complete freedom, and for instance permit mathematics and a foreign language, mathematics and philosophy etc.

On level B with mathematics as a minor, it would be possible to have one major (physics?) or two other minors.

Students taking level A examinations in mathematics would have their main interests in other fields. It is not necessary to discuss how these studies could be organized.

I said before that to each part of the mathematics course in school should correspond a larger course at the university. This may be going too far. If a teacher has a solid course in theory of matrices it must be possible for him to teach theory of determinants without a university course in that field. It is important that the teacher has at least had one complete course in a discipline of mathematics where he can see the whole system with definitions, axioms, theorems, deductions, proofs, etc. But some courses in schools are so specific, so different from others, that an equivalent course at the university is necessary. To be able to teach *calculus* in school you certainly need a calculus course at the university. The same is true for probability and statistics.

I think I have already mentioned a course which should be compulsory for all the three levels—a course on the structure of mathematics, on the working methods, etc. I have seen an American book with the following chapters which give you some ideas on what I have in mind: Language, Symbols, Compound

Statements, Arguments and Proofs, The Axiomatic Method, Introduction to Sets, Logic and Sets, The Structure of Sets, Number Sets, Conditions on Sets, Problem Solving, Relations, Functions, Counting, Probability.

Some of these chapters may be treated later in special courses, but a course of this kind should be a very good introduction to the study of mathematics, and also give the students ideas which they must have if they are to be *good* teachers. Such a foundation course should be taken by all future mathematics teachers, and *all* should further take a course in *history of mathematics*.

For teachers on level A, I would add two courses, one in geometry and one in algebra, giving the background for the more elementary teaching in these two fields. It should not be necessary to give detailed plans for these courses. In the algebra course we must have basic concepts of a set, phrases, sentences, equations, inequalities, numbers systems (rational, real, complex), absolute values, truth sets, graphs, etc.; in the geometry course figures defined as set of points, deduction and deductive theory in geometry, measurement, coordinate systems, transformations, geometric intuition, vectors, etc. What is important is to give teachers of mathematics on the first level a sure knowledge and insight in the subject matter to enable them to be good teachers. These courses should not just be a repetition of similar secondary school courses, but should go deeper and higher, and be richer and wider.

For a teacher on the B level, I would have the same three courses which I just mentioned, but also some more. It is not easy to measure such courses. Some students learn fast, others are slow learners. Some students cover two subjects at the same time, others work besides their studies. I dare say that at least one year of thorough study of mathematics is necessary for teachers on the B level, probably more: 1½ year.

These studies should comprise a course in linear algebra-sets, groups, rings, integral domains, fields of real and of complex numbers, linear equations, determinants, and matrices would be some of the chapters. Further, a course in calculus, especially giving the basic ideas of a function, sequences and series, limits,

derivation, the two integrals and some simple differential equations. A course in geometry must cover vectors, topology, transformations, foundations, may be some projective geometry and other disciplines.

Besides this, it should be possible to take some elective courses, for instance partial differential equations, probability and statistics, group theory, elementary number theory, numerical analysis, measurement and integral theory, rational mechanics, linear programming, game theory, etc.

A course in statistics and probability must be compulsory for all teachers in the level C. Here we should also have a course in mathematical analysis, analytic functions, advanced calculus, differential equations, functions of a complex variable, etc. Further, a course in number theory and maybe one in algebra, then studies in a rather large field selected by the student himself from which his special " homework "—thesis, problems, or whatever it may be, is taken. All courses should be strong and thorough, given by real scientists and mathematicians no matter whether at a university or at a special institution for future teachers.

I assume of course that no lecture is just a recitation from a textbook. The lectures should give hints, ideas and impulses. The students should be given the opportunity to make comments and raise questions. The examination papers should not only ask for giving back what is mechanically memorized, but should demand an independent understanding and mastery. During the studies exercises must be given and discussed. Without exercises no effective study!

*Today* we have several difficulties: 1) the shortage of mathematics teachers in schools, 2) the competition from industry, computers, etc. taking the best mathematicians paying them better, 3) the modernized school programs put new demands.

In some countries refresher courses have been introduced in the summer or also in the school year in the evenings or by correspondence.

Professor O. Ore said in a lecture: " A university training which is 20 years old, is too old if it is not supplemented and renewed ".

In the U.S.A. films have been prepared or will be made on probability, calculus, designed for mathematics teachers.

A short time ago a clever mathematics teacher said: " If only I could be given permission to go back to the university for half a year !"

We have a very important problem here. If we cannot retrain our mathematics teachers, say more than 30 years old, then we can have no reform in mathematics teaching in our schools !

In practically all countries, says the FIPESO-report, teachers in service have felt the need of refresher courses in order to keep pace with recent developments in the academic and pedagogical field. Attendance is optional, courses are often organized by the teachers associations.

The conclusion of the FIPESO report is this:

In-service training is a field where close co-operation between teachers and teachers' organizations on one hand and the authorities on the other must be regarded as indispensable in all countries. Let us hope that, in a not too distant future, an efficient in-service training, generously aided by the state, will be looked upon as an essential condition for the improvement of the school system of a country.

In the recommendations it was said:
Teachers attending such courses should receive adequate grants to cover their expenses.

3. The next field is the pedagogical or professional training, the " how " courses. As you know, they have not always been a part of the training of mathematics teachers, and even today this training does not seem to be compulsory in all countries.

I am quite convinced that it should be. Very few are " born " teachers, but most future teachers can profit by proper instructions, by professional training, and they can get insight into those many problems in elementary school mathematics where academic wisdom is not sufficient. The students must further study different textbooks (also from other countries), discuss examinations and tests used in mathematics, use of tables and instru-

ments, homework and many other problems which need to be discussed before actual teaching starts. Besides having these so-called methods courses, the students must know the school laws and regulations in the country, educational theory, history of education, educational psychology (learning, youth development, intelligence etc.) and hygiene, general didactics, elements of sociology, comparative education, audio-visual aids, programmed instruction. Further, they should follow teaching in school classes and teach there, being in the end inspected and given a mark, grade or paper indicating their teaching ability.

Instruction of this kind is mostly given in special institutions which can also be parts of a university. The teachers in these institutions must of course have experience from schools; they must be or must have been school teachers, with a strong background in their subject field.

The courses we find have different lengths—one semester, one year or more. We find a two-year course where professional instruction is combined with teaching. It is a trial period for which the young teacher is paid. In this case the academic training is supposed to be finished earlier. In other cases the academic and the professional training run parallel, which may be wise. On the other hand, I consider it best for the schools to get teachers—even student teachers—who by an academic degree can show that they really know mathematics.

Normally a one-year professional or pedagogical course should be sufficient. If the duration is only one semester, all young teachers ought to be given advisors, elder teachers who can assist them, and also if necessary, together with the principal and/or an inspector evaluate their work in school and thereby, if wanted, correct the first mark of teaching ability.

The pedagogical institutions will certainly be developed in the future and will be getting more tasks. They should not only train future teachers but also take up research work, try to find the best teaching methods, evaluate programmed instruction etc. Try to find out why some children have difficulties with mathematics, try to find out how children learn mathematical concepts, at what age they understand a proof and many other problems where psychology and mathematics must co-operate.

It must be possible to construct a much better organized and more effective mathematics teaching than we have to day.

4. As previously stated, there are four necessary conditions for being a good teacher. The last and fourth group is more difficult to define. We have here personal attributes and qualities which cannot easily be provoked or improved by instruction or teaching. What I have in mind is this: The good teacher must *understand* the students, must be able to follow them, be in contact with them, and must be *open* and *independent.* We have here for a great part natural gifts, but I have seen students change during their professional course, and I think more could be done. We need mathematics teachers who are open and understanding, who are not afraid of the textbook nor of their students.

To sum up, what I have said is this:

1. Good mathematics teachers must not only know their subject matter, but also have a wider general background in sciences as well as in humanities and in relationships in daily life.
2. They must be open and independent understand their students and their problems in mathematics and to a certain degree also outside mathematics.
3. Too many mathematics teachers need a *wider area of knowledge*, more adapted to the programmes in school, and they need a *deeper-going knowledge.* This should be given by mathematicians who not only give informative knowledge, but who at the same time inspire their students to active, independent work and creative invention, give them courage to use their imagination, and, on the whole, teach them in the *same way that these students are going to use later when they are teachers in class.*
4. The professional, pedagogical training must be compulsory in all countries. It can be improved, especially in many cases by discussing more than at is being done at present, all problems which may arise in mathematics classes in schools. The pedagogical institutions ought to start research work by experiments which could show the best teaching methods, and by taking up other problems in mathematics instruction.

The future mathematics teachers must be given sufficient time for student teaching (observations as well as teaching) and should in his first two years as a teacher be given assistance and guidance by elder, experienced teachers. Only after these two years, should a final mark or grade for his teaching ability be given.

The first five to ten years may give us trouble: we have too few teachers and we must retrain older teachers so that they can be able to teach the new programmes. But I am optimistic— I maintain that in ten years at most we shall have a sufficient number of well prepared, good mathematics teachers. But it will be necessary to have refresher courses for teachers in service. Let us remember that in Denmark just now 50 per cent of *all* mathematics teachers have taken a fortnight's summer course in modern mathematics. It must be a task for I.C.M.I. to find how such courses can be organized in the most effective way.