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Artikel: ON THE CONSTRUCTION OF RELATED EQUATIONS FOR THE ASYMPTOTIC THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS ABOUT A TURNING POINT

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The replacements which change T to T_ν are thus seen to be ones which replace

$$\lambda^{n-\nu}\left\{\delta_{p-\nu,j}+\frac{\sigma_{j,r}^{(p-\nu)}}{\lambda^r}\right\} \text{ by } \lambda^n\frac{\tau_\nu}{\lambda^r}.$$

It follows that

$$\frac{T_\nu}{T}=\lambda^\nu\frac{\theta_\nu(z,\lambda)}{\lambda^r},$$

with some function $\theta_\nu(z,\lambda)$ which is bounded over the z and λ domains. This gives to the relation (9. 3) the form

$$L^*(u) = l^*(m^*(u))-\frac{1}{\lambda^r}\sum_{\nu=1}^{p}\lambda^\nu\theta_\nu D^{p-\nu}m^*(u). \quad (9.7)$$

With the substitution of the expression for $D^{p-\nu}m^*(u)$, as it may be obtained from (4. 3) by writing $\bar{\gamma}_{i-s}$ in the place of γ_{i-s}, it is found that

$$L^*(u) = l^*(m^*(u))-\frac{1}{\lambda^r}\sum_{j=1}^{n}\lambda^j\omega_j(z,\lambda)D^{n-j}u, \quad (9.8)$$

with

$$\omega_j(z,\lambda) = \sum_{\nu=1}^{p}\sum_{s=0}^{p}\lambda^{-s}\binom{p-\nu}{s}\theta_\nu D^s\bar{\gamma}_{\mu-\nu-s}.$$

A comparison of this with the earlier result (6. 6) shows that

$$L^*(u) = L(u)-\frac{1}{\lambda^r}\sum_{j=1}^{n}\lambda^j\{\varepsilon_j(z,\lambda)+\omega_j(z,\lambda)\}D^{n-j}u. \quad (9.9)$$

The equation (9. 1), whose solutions are completely known, thus has coefficients which differ from those of the given equation (2. 1) only by terms that are of at least the r^{th} degree in $1/\lambda$. It is, therefore, by definition, a related equation.

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