

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 8 (1962)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE CONSTRUCTION OF RELATED EQUATIONS FOR THE ASYMPTOTIC THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS ABOUT A TURNING POINT

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Bibliographie

DOI: <https://doi.org/10.5169/seals-37963>

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The replacements which change T to T_ν are thus seen to be ones which replace

$$\lambda^{n-\nu} \left\{ \delta_{p-\nu, j} + \frac{\sigma_{j, r}^{(p-\nu)}}{\lambda^r} \right\} \text{ by } \lambda^n \frac{\tau_\nu}{\lambda^r}.$$

It follows that

$$\frac{T_\nu}{T} = \lambda^\nu \frac{\theta_\nu(z, \lambda)}{\lambda^r},$$

with some function $\theta_\nu(z, \lambda)$ which is bounded over the z and λ domains. This gives to the relation (9.3) the form

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{\nu=1}^p \lambda^\nu \theta_\nu D^{p-\nu} m^*(u). \quad (9.7)$$

With the substitution of the expression for $D^{p-\nu} m^*(u)$, as it may be obtained from (4.3) by writing $\bar{\gamma}_{i-s}$ in the place of γ_{i-s} , it is found that

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{j=1}^n \lambda^j \omega_j(z, \lambda) D^{n-j} u, \quad (9.8)$$

with

$$\omega_j(z, \lambda) = \sum_{\nu=1}^p \sum_{s=0}^p \lambda^{-s} \binom{p-\nu}{s} \theta_\nu D^s \bar{\gamma}_{\mu-\nu-s}.$$

A comparison of this with the earlier result (6.6) shows that

$$L^*(u) = L(u) - \frac{1}{\lambda^r} \sum_{j=1}^n \lambda^j \{ \varepsilon_j(z, \lambda) + \omega_j(z, \lambda) \} D^{n-j} u. \quad (9.9)$$

The equation (9.1), whose solutions are completely known, thus has coefficients which differ from those of the given equation (2.1) only by terms that are of at least the r^{th} degree in $1/\lambda$. It is, therefore, by definition, a related equation.

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