

<b>Zeitschrift:</b>	L'Enseignement Mathématique
<b>Herausgeber:</b>	Commission Internationale de l'Enseignement Mathématique
<b>Band:</b>	8 (1962)
<b>Heft:</b>	1-2: L'ENSEIGNEMENT MATHÉMATIQUE
<b>Artikel:</b>	ON THE CONSTRUCTION OF RELATED EQUATIONS FOR THE ASYMPTOTIC THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS ABOUT A TURNING POINT
<b>Autor:</b>	Langer, Rudolph E.
<b>Bibliographie</b>	
<b>DOI:</b>	<a href="https://doi.org/10.5169/seals-37963">https://doi.org/10.5169/seals-37963</a>

#### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 07.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

The replacements which change  $T$  to  $T_v$  are thus seen to be ones which replace

$$\lambda^{n-v} \left\{ \delta_{p-v, j} + \frac{\sigma_{j, r}^{(p-v)}}{\lambda^r} \right\} \text{ by } \lambda^n \frac{\tau_v}{\lambda^r}.$$

It follows that

$$\frac{T_v}{T} = \lambda^v \frac{\theta_v(z, \lambda)}{\lambda^r},$$

with some function  $\theta_v(z, \lambda)$  which is bounded over the  $z$  and  $\lambda$  domains. This gives to the relation (9.3) the form

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{v=1}^p \lambda^v \theta_v D^{p-v} m^*(u). \quad (9.7)$$

With the substitution of the expression for  $D^{p-v} m^*(u)$ , as it may be obtained from (4.3) by writing  $\bar{\gamma}_{i-s}$  in the place of  $\gamma_{i-s}$ , it is found that

$$L^*(u) = l^*(m^*(u)) - \frac{1}{\lambda^r} \sum_{j=1}^n \lambda^j \omega_j(z, \lambda) D^{n-j} u, \quad (9.8)$$

with

$$\omega_j(z, \lambda) = \sum_{v=1}^p \sum_{s=0}^p \lambda^{-s} \binom{p-v}{s} \theta_v D^s \bar{\gamma}_{\mu-v-s}.$$

A comparison of this with the earlier result (6.6) shows that

$$L^*(u) = L(u) - \frac{1}{\lambda^r} \sum_{j=1}^n \lambda^j \{ \epsilon_j(z, \lambda) + \omega_j(z, \lambda) \} D^{n-j} u. \quad (9.9)$$

The equation (9.1), whose solutions are completely known, thus has coefficients which differ from those of the given equation (2.1) only by terms that are of at least the  $r^{\text{th}}$  degree in  $1/\lambda$ . It is, therefore, by definition, a related equation.

#### REFERENCES

- [1] R. E. LANGER, The asymptotic solutions of ordinary linear differential equations of the second order with special reference to a turning point. *Trans. Amer. Math. Soc.*, vol. 67 (1949), pp. 461-490.
- [2] R. W. MCKELVEY, The solutions of second order linear ordinary differential equations about a turning point of order two. *Trans. Amer. Math. Soc.*, vol. 79 (1955), pp. 103-123.

- [3] R. E. LANGER, The solutions of a class of ordinary differential equations of the third order in a region containing a multiple turning point. *Duke Math. Jour.*, vol. 23 (1956), pp. 93-110.
- [4] N. D. KAZARINOFF, Asymptotic theory of second order differential equations with two simple turning points. *Archive f. Rat. Mech. and Anal.*, vol. 2 (1958), pp. 129-150.
- [5] R. E. LANGER, On the construction of related differential equations. *Trans. Amer. Math. Soc.*, vol. 81 (1956), pp. 394-410.
- [6] R. E. LANGER, Formal solutions and a related equation for a class of fourth order differential equations of hydrodynamic type. *Trans. Amer. Math. Soc.*, vol. 92 (1959), pp. 371-410.

Mathematics Research Center.

U.S. Army.

University of Wisconsin

Madison, Wis.

U.S.A.