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Artikel: ON THE CONSTRUCTION OF RELATED EQUATIONS FOR THE ASYMPTOTIC THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS ABOUT A TURNING POINT

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Kapitel: 3. The resultant.

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The relations (2. 5), (2. 6) and (2. 7) imply that $p+q=n$. We shall suppose the notation to be so assigned as to yield $p \leqq q$. It follows then also that

$$\sum_{i=0}^{k} b_i(z)\, c_{k-i}(z) = p_{k,0}(z)\,, \quad k = 0, 1, 2, \ldots, n\,, \tag{2. 8}$$

it being understood that the value 0 is to be assigned to any symbol $b_i(z)$ or $c_{k-i}(z)$ which, by virtue of its subscript, is not present in the formulas (2. 7).

A change of dependent variable can be applied to the equation (1. 1) to give it a form for which the coefficient $b_1(z)$ is identically zero. Considerable simplifications of the formulas result therefrom. We shall not resort to that normalization, however, refraining from it in order to keep the roles of the factors $B(\chi)$ and $\Gamma(\chi)$ interchangeable.

3. The resultant.

It is a hypothesis that the two polynomials (2. 7) are relatively prime over the z-region. If we denote their resultant by $\varDelta(z)$, we have accordingly

$$\varDelta(z) \neq 0\,, \tag{3. 1}$$

with

$$\varDelta(z) \equiv \left[\begin{array}{cccccccccccccccc}
1 & b_1 & b_2 & - & - & - & - & b_p & 0 & - & - & - & - & - & - & 0 \\
0 & 1 & b_1 & b_2 & - & - & - & - & b_p & 0 & - & - & - & - & - & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
0 & - & - & - & - & - & - & - & 0 & 1 & b_1 & - & - & - & - & b_p \\
1 & c_1 & c_2 & - & - & - & - & - & - & - & - & c_q & 0 & - & - & 0 \\
0 & 1 & c_1 & - & - & - & - & - & - & - & - & - & c_q & 0 & - & 0 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
0 & - & - & - & - & 0 & 1 & c_1 & c_2 & - & - & - & - & - & - & c_q
\end{array}\right] \tag{3. 2}$$

We shall find use for the relation (3. 2). However, for future use we find it convenient to formulate the relative primary of the factors $B(\chi)$ and $\Gamma(\chi)$ also in an alternative form. This is done as follows:

Let K designate the familiar matrix

$$K(z) = \begin{bmatrix} 0 & 0 & - & - & - & - & -b_p \\ 1 & 0 & - & - & - & - & -b_{p-1} \\ 0 & 1 & 0 & - & - & - & -b_{p-2} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ 0 & - & - & - & - & 1 & -b_1 \end{bmatrix} \tag{3.3}$$

The eigen-values of this are the roots x_i, $i = 1, 2, \ldots, p$ of the equation $B(x) = 0$. If we designate by ξ_i an eigen-vector corresponding to x_i we have

$$K^h \xi_i = x_i^h \xi_i, \quad h = 1, 2, \ldots, q,$$

and accordingly

$$\Gamma(K) \xi_i = \Gamma(x_i) \xi_i .$$

Thus $\Gamma(x_i)$ is an eigenvalue of the matrix $\Gamma(K)$, and, since the product of the eigenvalues is the determinant of the matrix, we have

$$|\Gamma(K)| = \prod_{i=1}^{p} \Gamma(x_i) .$$

Observing that no factor on the right is zero, and giving to $\Gamma(K)$ its explicit form, we conclude with the result

$$\left| \sum_{j=0}^{q} c_j(z) K^{q-j}(z) \right| \neq 0 \tag{3.4}$$

4. Two differencial operators of the orders p and q.

Let the functions $\beta_j(z, \lambda)$ and $\gamma_i(z, \lambda)$ be taken to be polynomials of the degree $(r - 1)$ in $1/\lambda$, thus

$$\beta_j(z, \lambda) = \sum_{\nu=0}^{r-1} \frac{\beta_{j,\nu}(z)}{\lambda^\nu}, \quad \beta_{j,0}(z) \equiv b_j(z); \; j = 1, 2, \ldots, p,$$

$$\gamma_i(z, \lambda) = \sum_{\nu=0}^{r-1} \frac{\gamma_{i,\nu}(z)}{\lambda^\nu}, \quad \gamma_{i,0}(z) \equiv c_i(z); \; i = 1, 2, \ldots, q. \tag{4.1}$$