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The relations (2. 5), (2. 6) and (2. 7) imply that $p+q = n$. We shall suppose the notation to be so assigned as to yield $p \leq q$. It follows then also that

$$\sum_{i=0}^k b_i(z) c_{k-i}(z) = p_{k,0}(z), \quad k = 0, 1, 2, \dots, n, \quad (2.8)$$

it being understood that the value 0 is to be assigned to any symbol $b_i(z)$ or $c_{k-i}(z)$ which, by virtue of its subscript, is not present in the formulas (2. 7).

A change of dependent variable can be applied to the equation (1. 1) to give it a form for which the coefficient $b_1(z)$ is identically zero. Considerable simplifications of the formulas result therefrom. We shall not resort to that normalization, however, refraining from it in order to keep the roles of the factors $B(\chi)$ and $\Gamma(\chi)$ interchangeable.

3. THE RESULTANT.

It is a hypothesis that the two polynomials (2. 7) are relatively prime over the z -region. If we denote their resultant by $\Delta(z)$, we have accordingly

$$\Delta(z) \neq 0, \quad (3.1)$$

with

$$\Delta(z) \equiv \begin{bmatrix} 1 & b_1 & b_2 & - & - & - & b_p & 0 & - & - & - & - & - & 0 \\ 0 & 1 & b_1 & b_2 & - & - & - & b_p & 0 & - & - & - & - & 0 \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ 0 & - & - & - & - & - & - & 0 & 1 & b_1 & - & - & - & b_p \\ 1 & c_1 & c_2 & - & - & - & - & - & - & - & c_q & 0 & - & 0 \\ 0 & 1 & c_1 & - & - & - & - & - & - & - & - & c_q & 0 & 0 \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ 0 & - & - & - & - & 0 & 1 & c_1 & c_2 & - & - & - & - & c_q \end{bmatrix} \quad (3.2)$$

We shall find use for the relation (3. 2). However, for future use we find it convenient to formulate the relative primary of

the factors $B(x)$ and $\Gamma(x)$ also in an alternative form. This is done as follows:

Let K designate the familiar matrix

$$K(z) = \begin{bmatrix} 0 & 0 & - & - & - & - & -b_p \\ 1 & 0 & - & - & - & - & -b_{p-1} \\ 0 & 1 & 0 & - & - & - & -b_{p-2} \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ 0 & - & - & - & - & 1 & -b_1 \end{bmatrix} \quad (3.3)$$

The eigen-values of this are the roots x_i , $i = 1, 2, \dots, p$ of the equation $B(x) = 0$. If we designate by ξ_i an eigen-vector corresponding to x_i we have

$$K^h \xi_i = x_i^h \xi_i, \quad h = 1, 2, \dots, q,$$

and accordingly

$$\Gamma(K) \xi_i = \Gamma(x_i) \xi_i.$$

Thus $\Gamma(x_i)$ is an eigenvalue of the matrix $\Gamma(K)$, and, since the product of the eigenvalues is the determinant of the matrix, we have

$$|\Gamma(K)| = \prod_{i=1}^p \Gamma(x_i).$$

Observing that no factor on the right is zero, and giving to $\Gamma(K)$ its explicit form, we conclude with the result

$$\left| \sum_{j=0}^q c_j(z) K^{q-j}(z) \right| \neq 0 \quad (3.4)$$

4. TWO DIFFERENTIAL OPERATORS OF THE ORDERS p AND q .

Let the functions $\beta_j(z, \lambda)$ and $\gamma_i(z, \lambda)$ be taken to be polynomials of the degree $(r-1)$ in $1/\lambda$, thus

$$\beta_j(z, \lambda) = \sum_{v=0}^{r-1} \frac{\beta_{j,v}(z)}{\lambda^v}, \quad \beta_{j,0}(z) \equiv b_j(z); \quad j = 1, 2, \dots, p,$$

$$\gamma_i(z, \lambda) = \sum_{v=0}^{r-1} \frac{\gamma_{i,v}(z)}{\lambda^v}, \quad \gamma_{i,0}(z) \equiv c_i(z); \quad i = 1, 2, \dots, q. \quad (4.1)$$