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In short, the turning index of σ^{n-1} under the assumption of the absence of fixed points is non-zero, a fact which contradicts Lemma 4. Hence f has at least one fixed point, and Theorem 2 is proved.

COROLLARY 1. *Let E^n be a closed solid n -sphere and f a continuous mapping of E^n into R^n such that f maps the boundary S^{n-1} of E^n into E^n . Then f has at least one fixed point.*

Proof. If no point of S^{n-1} is fixed, then the hypotheses of Theorem 2 are seen to be satisfied with e at the center of the sphere E^n and $\alpha = 0$.

Clearly, Corollary 1 also follows immediately from Theorem 1. Proofs of this corollary also appear in the literature ([3], page 115).

COROLLARY 2. *Let $\eta^n \subset R^n$ be a closed n -cell with boundary σ^{n-1} , and f and g two continuous maps of η^n into R^n such that for no point $\sigma \in \sigma^{n-1}$ is $f(\sigma) = g(\sigma)$. If there exists an inner point e of η^n and a constant angle β , $0 \leq \beta \leq \pi$, such that for no point $\sigma \in \sigma^{n-1}$ is β an angle between the vectors e, σ and $f(\sigma), g(\sigma)$, then there is a point $\eta_0 \in \eta^n$ such that $f(\eta_0) = g(\eta_0)$.*

Proof. Consider the map h of η^n into R^n such that for every point $\eta \in \eta^n$ the vectors $\eta, h(\eta)$ and $f(\eta), g(\eta)$ are equal. By Theorem 2, the map h has a fixed point η_0 . Consequently, $f(\eta_0) = g(\eta_0)$.

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