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$R^n$ , and  $(n-1)$ -spheres oriented with orientations induced by their interiors.

Symbols  $c^{n-1}$ ,  $g^{n-1}$ , ... denote oriented  $(n-1)$ -cycles in  $R^n$ ;  $D^{n-1}$ ,  $V^{n-1}$ , ... denote  $(n-1)$ -spheres in  $R^n$ .  $E^n$  denotes a closed solid  $n$ -sphere in  $R^n$ , and the boundary of  $E^n$  is denoted by  $S^{n-1}$ .  $\eta^n$  denotes a closed  $n$ -cell in  $R^n$  and the boundary of  $\eta^n$  is denoted by  $\sigma^{n-1}$ .

In this paper  $\eta^n$  is assumed to be the image of  $E^n$  under homeomorphism  $\theta$ , and  $\eta^n$  and  $\sigma^{n-1}$  obtain their orientations from  $E^n$  and  $S^{n-1}$  respectively.

### 3. THE TURNING INDEX

Let  $c^{n-1}$  be an  $(n-1)$ -cycle in  $R^n$  and  $g$  a continuous map of  $c^{n-1}$  into  $R^n$  having no fixed point. Let  $D^{n-1}$  be an  $(n-1)$ -sphere with center 0, called a *direction sphere* [2]. Let  $c^{n-1}$  be mapped on  $D^{n-1}$  as follows. To a point  $c \in c^{n-1}$  there corresponds a point  $d \in D^{n-1}$  such that the line segment from 0 to  $d$  has the same sense and direction as that from  $c$  to  $g(c)$ . The resulting  $(n-1)$ -cycle  $g^{n-1}$  on  $D^{n-1}$  is called, in the sequel, *the  $(n-1)$ -cycle  $g^{n-1}$  resulting from  $g$  applied to  $c^{n-1}$* , and the degree of the resulting map, that is, the multiple of  $D^{n-1}$  which is homologous to  $g^{n-1}$  (which is clearly independent of the radius of  $D^{n-1}$  and the location of 0) is called the *turning index* of  $c^{n-1}$  under  $g$ .

If  $p$  is a point not on  $c^{n-1}$ , the *index of  $p$  relative to  $c^{n-1}$*  is defined as the turning index of the map which maps every point of  $c^{n-1}$  into  $p$ . (For odd  $n$ , this is the negative of the corresponding definition given in [3], as shown by Theorem 1.5, page 105).

### 4. PRELIMINARY LEMMAS

LEMMA 1. *Let  $g$  and  $h$  be two continuous maps into  $R^n$  of an  $(n-1)$ -cycle  $c^{n-1}$ , such that neither leaves any point of  $c^{n-1}$  fixed, and, for no point  $c \in c^{n-1}$  are the directions from  $c$  to  $g(c)$  and from  $c$  to  $h(c)$  exactly opposite. Then the turning indices of  $c^{n-1}$  under  $g$  and  $h$  are equal.*

*Proof.* For each  $c \in c^{n-1}$ , the directions of the two vectors  $c, g(c)$  and  $c, h(c)$  are not opposite and hence, if not identical,