Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 7 (1961)

Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE COHOMOLOGY ALGEBRA OF A SPACE

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Bibliographie

DOI: https://doi.org/10.5169/seals-37129

preted as primary cohomology operations, and the elements of $H^*(D; Z_p)$ as secondary operations defined on cohomology classes annihilated by y (see [1]). Numerous non-trivial secondary operations have been found.

Thus to realize W as the cohomology algebra of a space, we must modify D so as to eliminate the unwanted elements of $H^*(D; Z_p)$. But before trying this, we should reexamine our objective. It was to construct a space whose cohomology has a single generator and is maximal subject to a single relation. In one sense D already satisfies our requirement. If we admit *secondary* cohomology operations as well as the primary operations $\mathscr{A}_p$, then the g^*-image of the generator of $H^*(K(Z_p, q); Z_p)$ does in fact generate $H^*(D; Z_p)$, and the latter is free in the sense that there are no accidental relations. This is a restatement of the identification of elements of $H^*(X; Z_p)$ with secondary operations associated with y.

Thus, in attempting to realize W, we have tacitly assumed that we know what is meant by " one generator subject to one relation ". Our prejudices have again interposed themselves. The correct procedure is to analyse fully the structure of $H^*(D; Z_p)$, and then we may know how to define the concept of one generator subject to one relation.

Eventually we will want to know how to describe algebraically the cohomology algebra on k generators subject to r_1 primary relations, r_2 secondary relations, etc. We know already how to realize this algebra using Eilenberg-MacLane complexes and the fibre space constructions of Postnikov [16]. But we are a long way from being able to describe the algebra in direct algebraic terms.

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