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proof which possibly goes more to the heart of the matter emerges from the point of view of Atiyah and Hirzebruch. They define the groups:

$$KU^i(X) = \pi[E^{-i}X; Z \times B_U] \quad i \leq 0$$

where  $\pi[A, B]$  denotes homotopy classes of maps. In this terminology the periodicity formula:  $\Omega^2(Z \times B_U) = Z \times B_U$  is expressed by:

$$KU^i(X) = KU^{i+2}(X) \quad i \leq -2.$$

Now Atiyah and Hirzebruch use this recurrence to define  $KU^i(X)$  for all integers  $i$ , and then show that the resulting functor  $X \rightarrow \{KU^i(X)\}$  satisfies all the axioms of a cohomology theory—except for the dimension axiom. Further they observe that the uniqueness theorem of Eilenberg-Steenrod can be generalized to yield a spectral sequence relating  $E_2 = H^*(X; KU^*(p))$  to  $KU^*(X)$ . (Here  $KU^i(p)$ —the  $KU$ —theory of a point—is  $Z$  if  $i$  is even and  $0$  otherwise.) This sequence immediately implies the proposition. (See [8].)

To return to our bundles  $\xi^m$  on  $P_n$ . By the proposition just discussed the restriction of  $\xi^m$  to  $P_{n-k}$  will be trivial if  $m \geq n - k$ . By trivializing this element on  $P_{n-k}$  we obtain bundles  $\xi^m$  on  $P_{n,k}$  which under the projection  $\pi: P_n \rightarrow P_{n,k}$  go over into  $\xi^m$ ,  $m \geq n - k$ . In particular,  $\pi^* ch(\xi^m) = (e^x - 1)^m$ . Thus in any case we obtain these criteria the  $S$ -reducibility of  $P_{n,k}$ :

$P_{n,k}$  is  $S$ -reducible only if the coefficient of  $x^{n-1}$  in  $(e^x - 1)^m$ ,  $m \geq n - k$ , is an integer.

This is the number theoretical condition from which Atiyah and Todd deduce theorem III. Their result is the best possible one obtainable from the test-space  $B_U$ , because one can show quite easily, with the spectral sequence alluded to earlier, that the elements  $1, \xi^m; n - 1 \geq m \geq n - k$ ; form a base of  $KU(P_{n,k})$ .

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