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proof which possibly goes more to the heart of the matter emerges from the point of view of Atiyah and Hirzebruch. They define the groups:

$$KU^i(X) = \pi[E^{-i}X;\ Z \times B_U] \qquad i \leqq 0$$

where $\pi[A, B]$ denotes homotopy classes of maps. In this terminology the periodicity formula: $\Omega^2(Z \times B_U) = Z \times B_U$ is expressed by:

$$KU^i(X) = KU^{i+2}(X) \qquad i \leqq -2 .$$

Now Atiyah and Hirzebruch use this recurrence to define $KU^i(X)$ for all integers $i$, and then show that the resulting functor $X \to \{KU^i(X)\}$ satisfies all the axioms of a cohomology theory —except for the dimension axiom. Further they observe that the uniqueness theorem of Eilenberg-Steenrod can be generalized to yield a spectral sequence relating $E_2 = H^*(X; KU^*(p))$ to $KU^*(X)$. (Here $KU^i(p)$—the $KU$—theory of a point—is $Z$ if $i$ is even and 0 otherwise.) This sequence immediately implies the proposition. (See [8].)

To return to our bundles $\xi^m$ on $P_n$. By the proposition just discussed the restriction of $\xi^m$ to $P_{n-k}$ will be trivial if $m \geqq n-k$. By trivializing this element on $P_{n-k}$ we obtain bundles $\xi^m$ on $P_{n,k}$ which under the projection $\pi$: $P_n \to P_{n,k}$ go over into $\xi^m$, $m \geqq n-k$. In particular, $\pi^* ch(\xi^m) = (e^x - 1)^m$. Thus in any case we obtain these criteria the $S$-reducibility of $P_{n,k}$:

$P_{n,k}$ is S-*reducible only if the coefficient of* $x^{n-1}$ *in* $(e^x - 1)^m$, $m \geqq n-k$, *is an integer.*

This is the number theoretical condition from which Atiyah and Todd deduce theorem III. Their result is the best possible one obtainable from the test-space $B_U$, because one can show quite easily, with the spectral sequence alluded to earlier, that the elements 1, $\xi^m$; $n-1 \geqq m \geqq n-k$; form a base of $KU(P_{n,k})$.

## BIBLIOGRAPHY

[1] ADAMS, J., On the nonexistence of elements of Hopf invariant one. *Ann. of Math.*, 72 (1960), 20-104.
[2] ATIYAH, M., *Thom Complexes* (to be published).

[3] ATIYAH, M., and F. HIRZEBRUCH, Riemann-Roch theorems for differentiable manifolds. *Bull. Amer. Math. Soc.*, 65 (1959), 276-281.

[4] —— and J. TODD, On complex Stiefel Manifolds. *Proc. Camb. Phil.* Soc. (1960).

[5] BOTT, R., The stable homotopy of the classical groups. *Ann. of Math.*, 70 (1959).

[6] —— Quelques remarques sur les théorèmes de périodicité. *Bull. Soc. Math. France* (1959), 293-310.

[7] —— and J. MILNOR, On the parallelizability of the spheres. *Bull. Amer. Math. Soc.*, 64 (1958), 87-89.

[8] ECKMANN, B., Gruppentheoretischer Beweis des Satzes von Hurwitz-Radon über die Komposition quadratischer Formen. *Comm. Math. Helv.*, 15 (1943), 358-366.

[9] HIRZEBRUCH, F., A Riemann-Roch theorem for differentiable manifolds. *Séminaire Bourbaki* (1959), n° 177.

[10] HURWITZ, A., Ueber die Komposition des quadratischen Formen. *Math. Ann.*, 88 (1923), 1-25.

[11] —— Ueber die Komposition quadratischer Formen von beliebig vielen Variabeln. *Nachr. Ges. d. Wiss. Göttingen* (1898), 309-316.

[12] JAMES, I., The intrinsic join. *Proc. London Math. Soc.* (3), 8 (1958), 507-35.

[13] —— Cross sections of Stiefel manifolds. *Ibid.*, 536-47.

[14] —— Spaces associated with Stiefel Manifolds. *Ibid.* (3), 9 (1959), 115-40.

[15] KERVAIRE, M., Nonparallelizability of the $n$-sphere for $n > 7$. *Proc. Nat. Acad. Sci. U.S.A.* (1958), 283-283.

[16] RADON, J., Lineare Scharen orthogonaler Matrizen. *Abh. Sem. Hamburg*, I (1923), 1-14.

[17] SPANIER, E. and J. H. C. WHITEHEAD, Duality in homotopy theory. *Mathematica*, 2 (1955), 56-80.

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