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To verify this we need slight extensions of Lemma 2, and of the fact that derivatives possess the Darboux property.

LEMMA 2'. If f is continuous and f' exists (finite or infinite) in (p, q) , while $f(p^+)$ does not exist (finite or infinite); if G is continuous in $[p, q)$, G' exists (finite) in (p, q) and $G'(x) \neq 0$ in (p, q) ; then $f'(x)/G'(x)$ assumes every finite value in (p, q) .

Since $f(p^+)$ does not exist, $H(x) = f(x) - \lambda G(x)$ does not approach a limit (since $G(p^+)$ does exist). Hence H is not monotonic and so possesses extrema. At an extremum ξ we have $H'(\xi) = 0$, so $f'(\xi) = \lambda G'(\xi)$. Since $G'(\xi)$ is neither 0 nor infinite, $f'(\xi)/G'(\xi) = \lambda$.

LEMMA 3. If f and G are continuous in $[p, q]$; if f' exists (finite or infinite) in $[p, q]$, and G' exists (finite) in $[p, q]$; if $f'(p)$ and $f'(q)$ are finite and G' has a fixed sign (and hence is never 0) in $[p, q]$; and if

$$f'(p)/G'(p) < c < f'(q)/G'(q),$$

then there is a ξ in (p, q) such that $f'(\xi)/G'(\xi) = c$.

This says in effect that f'/G' , like f' , has the Darboux property.

Consider $H(x) = f(x) - cG(x)$ and suppose for definiteness that $G'(p) > 0$. Then $H'(p) < 0$, $H'(q) > 0$, so the continuous function H cannot assume its minimum at p or q . If H assumes its minimum at ξ , we have $f'(\xi) = cG'(\xi)$ and so (since $G'(\xi)$ is neither zero nor infinite), $f'(\xi)/G'(\xi) = c$.

It now follows just as before that Theorems 1 and 2 hold, with $g = 1/G'$ in (2).

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