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Autor: Carlitz, L.

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As in the case n even, it will suffice to take $r \equiv 3 \pmod 4$. Let 2^{e_r} denote the highest power of 2 dividing $r+1$, and consider $2^{-e_r}T$. For $e_r \leqslant 2$, we get the exponent -1; for $e_r \geqslant 3$ it follows that $\frac{1}{2}(r-1) \geqslant e_r$. Hence if $k+2 < e_r$, we get the exponent $k+2-e_r$, while if $k+2 \geqslant e_r$ we again get -1 (at most). Consequently

$$\Delta_n \equiv 0 \pmod{2^{3n-e+k+2}} \quad (k+2 < e_r)\ , \tag{16}$$

$$\Delta_n \equiv 0 \pmod{2^{3n-1}} \quad (k+2 \geqslant e_r)\ . \tag{17}$$

Comparing (16) and (17) with (13) and (14) we may accordingly state the following

THEOREM. *Let* $2^e \leqslant 2n < 2^{e+1}$ *and let* 2^k *denote the highest power of 2 dividing* n *or* $n-1$ *according as* n *is even or odd. Then* Δ_n *as defined by (1) and (3) satisfies (16) and (17).*

Applying the Staudt-Clausen theorem, it follows from (3) that

$$p\Delta_n \equiv -\sum_{r>0} (-1)^{n-rm} \binom{n}{rm} \pmod p\ ,$$

where $p = 2m+1$ is an odd prime. It can be shown that

$$\sum_{r>0} (-1)^{n-rm} \binom{n}{rm} \equiv \begin{cases} 2 & (p-1 \mid n) \\ \binom{m}{k} & (p-1 \mid n+k,\ 0 < k \leqslant m) \\ 0 & \text{otherwise.} \end{cases}$$

In particular the prime factors of the denominator of Δ_n are simple and cannot exceed $2n+1$.

In connection with formula (4) above it may be of interest to cite the formula [2, p. 189]

$$(-1)^n \sum_{r=0}^{n} \binom{n}{r} \frac{B_{n+r+1}}{n+r+1} + \frac{n!\,n!}{2\,(2n+1)!} = 0\ .$$

REFERENCES

1. G. FROBENIUS, Uber die Bernoullischen Zahlen und die Eulerschen Polynome, *Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1910), pp. 809-847.
2. N. NIELSEN, *Traité élémentaire des nombres de Bernoulli.* Paris, 1923.
3. N. E. NÖRLUND, *Vorlesungen über Differenzenrechnung.* Berlin, 1924.

Duke University.