Philosophy of Instruction.

The aims of mathematical instruction, the methods and materials used in instruction, and the proposed reforms and trends in the various countries show great contrasts as well as universal movements. These pedagogical phases are reflected in two outstanding view points which we can label (1) mathematics for the better life, i.e. for its intrinsic value, or for its own sake and (2) mathematics for a better living, i.e. for its application to science, technology, and social problems that will result in more efficient practical day by day living. Thus, for one country the goal of mathematics instruction is stated as follows:

" The pupils must learn to compute knowingly, rationally, and quickly both in written and oral work; to use arithmetic to solve practical problems and to answer questions; to develop logical thinking, initiative and creative powers; to acquire logical and systematic concepts of space; to solve practical exercises essential for use in polytecnical study."

Another country reports that the purpose of teaching mathematics is:

" To develop basic knowledge and lay the foundation for further study. In arithmetic, problems are preferentially applied to the industrial and technical sphere; in algebra, the start is with concrete problems and principles that can be applied to the solution of equations; operations in algebra are by functions — first experimentally and numerically — followed by logical elementary proofs of theorems. This simplifies the transition to deductive geometry. In the elementary school inductive methods are used, namely observation and description, making of models, abstraction of properties, formulation of definitions and laws and finally logical models, the latter however not in perfection."

The opinion that teaching in mathematics must begin with concrete physical things, the study of which, by inductive methods, leads to certain abstractions from which a mathematical system can be built, prevails all over the world. That arithmetic could be taught without the use of rather elaborate equipment — beads, abaci, number charts, colored pegs, Mon-

tessori materials, and so on ad infinitum — is an alien thesis to present day elementary mathematics instruction. So it appears that the mathematical education of youth must reflect the historical development of mathematics — i.e.: there must be a period of informal experience — then a fumbling of explaining the environment — out of which fundamental concepts emerge, are clarified and refined — then a formalization of the mathematics with memorization and application.

In the spirit of this theory the instruction during the first four or six years is guided by a predominance of psychology of learning in which more attention is given to the child's learning ability and his social needs than to the subject matter. In many countries however, just before the examination for admittance to the secondary program, the only social need becomes the learning of sufficient skills and tricks for passing the examination. Thereafter, in the secondary school the subject matter of mathematics and its gradual axiomatizing becomes the predominant factor. Since the child's ability to learn is not considered, a large and continuous rate of failure persists in the secondary school.

A glance at most textbooks for the ages 6 to 15 years reveals the startling fact that there is a tremendous amount of repetition of previous material, repeated in the same dull spirit as originally presented several years prior. Evidently no one expects a child to have learned and remembered the material taught in previous years. It is rather encouraging then to see a few countries taking a decided stand against this stultifying method by saying: "The work of the secondary school is not to repeat the study of the elementary school but to base its teaching and build new knowledge on the mathematics previously learned." If our students were expected to know what they had been taught — the material on which they had passed examinations the year before, there might be a resurrection of student interest in mathematical study, and in teaching methods, that could well border on the miraculous.

Before we turn to some promising trends in the teaching of elementary arithmetic, there is one instructional feature that all countries insist upon — namely Mental Arithmetic. How-

ever, the concept of mental arithmetic is not the same for all countries. Whatever it is, almost universally the use and stress on mental arithmetic begins in the second year of elementary school and continues, usually with daily (or at least periodic) drills, right through the age of 15 years. The one concept of mental arithmetic that is predominant is that of rapid calculation without the use of paper or pencil. Short cuts and tricks are learned (sometimes rationalized) but the purpose is to save time for later mathematics. Speed is of the essence and of course accuracy is demanded. The second concept extends beyond computation, to problem solving, allowing the use of the basic structure of the decimal system and its laws of operation, for the mental estimation, approximation, and exact solution of problems as well as for checking. Its emphasis is on thinking — reasoning, and understanding and not on speed. This concept offers power to the initiative and creativity of pupils learning, as well as interest and challenge in the subject, and it in no way deters from speed for those pupils who are capable.

Some Trends.

All countries are engaged in studying their mathematics education. A few countries are engaged in systematic experiment, but most study is made by scattered efforts of a few leaders or interested persons. Whether by parental pressure, experiment, or changing cultural patterns, there has been a gradual shift from mere rote — manipulative teaching of arithmetic, through complicated computational exercises, to the teaching by rationalization of the fundamental concepts and laws underlying the operations on number, including the decimal system of notation. Such a shift can be looked upon only with favor by those interested in the mathematical knowledge of our future society.

The result is that the work of the first four or six years is no longer regarded as reckoning or arithmetic, but as mathematics and is being labelled as such in the schools. The one drawback to the rapid promotion of this 'rational' point of view is the lack of knowledge of the elementary school teacher