Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 4 (1958)

Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE NORM OF A REAL LINEAR TRANSFORMATION IN MINKOWSKI SPACE

Autor: Taylor, Angus E.

Kapitel: 3. CONCLUDING REMARKS

DOI: https://doi.org/10.5169/seals-34630

Then, at (0,1),

$$P_1 = p \sum_k c_k^2 \alpha_k \beta_k , \quad P_2 = p \sum_k c_k^2 \beta_k^2 ,$$
$$P = \sum_k c_k^2 (\alpha_k^2 + \beta_k^2) .$$

Now, since $c_k \neq 0$ and the vectors $(\alpha_1, ..., \alpha_n)$, $(\beta_1, ..., \beta_n)$ are linearly independent, the vectors $(c_1 \alpha_1, ..., c_n \alpha_n)$ and $(c_1 \beta_1, ..., c_n\beta_n)$ are also linearly independent. Consequently, by Cauchy's inequality,

$$|P_1| < p \Big(\sum_k c_k^2 \alpha_k^2\Big)^{1/2} \Big(\sum_k c_k^2 \beta_k^2\Big)^{1/2} ,$$

the inequality being strict because of the linear independence. Consequently

$$P_1^2 < p P_2 \sum_k c_k^2 \alpha_k^2 .$$

Then

$$P_1^2 + P_2^2 < p P_2 \sum_k c_k^2 \alpha_k^2 + p P_2 \sum c_k^2 \beta_k^2 .$$

The right side here is exactly $p P_2 P$, and so the proof of (8) is complete. This finishes the proof of the lemma. We have already pointed out how the lemma leads to a proof of the theorem.

3. Concluding remarks

In conclusion, we point out that the results we have described were known to M. Riesz when he wrote his paper on convexity and bilinear forms.[1] He made a brief sketch of the arguments in support of the results. But the intended form of Riesz's argument has seemed obscure to some people, and the results themselves are apparently not much known outside the circle of those who are thoroughly familiar with Riesz's paper. Hence it has seemed to be worth while to emphasize the results and to put the details of the proof on record.

The University of California, Los Angeles.

[1] M. Riesz, *Sur les maxima des formes bilinéaires et sur les fonctionelles linéaires*, Acta Math. 49, 465-497 (1927).