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THE NORM OF A REAL LINEAR TRANSFORMATION IN MINKOWSKI SPACE

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(Reçu le 27 février 1958)

1. THE DEFINITION OF NORM

By the Minkowski space $l^p(n)$ we mean the space of vectors $x = (\xi_1, \dots, \xi_n)$ with the norm of x defined by

$$\|x\|_p = \left(\sum_i |\xi_i|^p \right)^{1/p}.$$

Here it is supposed that $p \geq 1$, so that $\|x\|_p$ is a norm on $l^p(n)$.

If $l^p(n)$ and $l^q(m)$ are Minkowski spaces of dimensions n and m , respectively, a linear transformation A of $l^p(n)$ into $l^q(m)$ is determined by a matrix (a_{jk}) of constants ($j = 1, \dots, m, k = 1, \dots, n$); if A transforms x into $y = (\eta_1, \dots, \eta_m)$, the η 's are given in terms of the ξ 's by the equations

$$\eta_j = \sum_{k=1}^n a_{jk} \xi_k \quad j = 1, \dots, m.$$

If we write $y = Ax$, the norm of A is defined as

$$\|A\| = \max_{x \neq 0} \frac{\|Ax\|_q}{\|x\|_p} = \max_{x \neq 0} \frac{\left(\sum_j \left| \sum_k a_{jk} \xi_k \right|^q \right)^{1/q}}{\left(\sum_k |\xi_k|^p \right)^{1/p}}.$$

We may consider all of these things with respect to the complex field, letting the vector components ξ_1, \dots, ξ_n and the

matrix elements a_{jk} be complex numbers. In this case we call $l^p(n)$ a *complex* Minkowski space and A a *complex* linear transformation. But we may equally well confine our attention to real scalars, in which case the space and the transformation are called *real*.

Now, if A is a transformation determined by a matrix of real elements a_{jk} , the transformation can be considered either as a real transformation or as a complex transformation, and accordingly there are two possible definitions of its norm. If in (1) we allow x to vary over all nonzero elements of the complex space $l^p(n)$, we get the norm of A as a complex transformation, whereas if we restrict the vector x to have real components, we get the norm of A as a real transformation. Let us denote these two norms by

$$\|A\|_c \quad \text{and} \quad \|A\|_r.$$

2. THE THEOREM

We shall prove the following result:

THEOREM *Let A be a transformation of $l^p(n)$ into $l^q(m)$ determined by a matrix (a_{jk}) of real constants. Suppose $q \geq p \geq 1$. Then*

$$\|A\|_c = \|A\|_r. \quad (2)$$

Proof. We first observe that when p is fixed and q varies subject to $q \geq p$, $\|A\|$ is a continuous function of q at $q = p$, regardless of whether we have a real or a complex transformation. For, let the dependence on q be exhibited by writing $\|A\| = M(q)$. It is well known that $\|y\|_q$ does not decrease as q decreases. Hence $p < q$ implies

$$\frac{\|Ax\|_q}{\|x\|_p} \leq \frac{\|Ax\|_p}{\|x\|_p},$$

whence also $M(q) \leq M(p)$. Now suppose that x is chosen ($x \neq 0$) so that

$$\frac{\|Ax\|_p}{\|x\|_p} = M(p).$$