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RANGE OF MATHEMATICAL EDUCATION IN SWEDISH GRAMMAR SCHOOLS WITH REGARD TO THE EXAMINATION-PAPER

BY

O. FROSTMAN.

At the congress in Amsterdam I had the opportunity to give you a brief report of the organization of the top forms of the Swedish Grammar School, the " Gymnasium ", as we call it, attended by pupils of about 15-19 years of age. To understand the problems which I want to discuss today it will be necessary for me to give a short recapitulation of some details of this organization. The " Gymnasium " is divided into three different sides, of which henceforth only the science side will be dealt with. During the last two scholar years the science side is again divided into two branches, the biological respectively the mathematical branch, of which the latter has the most extensive course in mathematics. This course comprises: *Arithmetic and elementary algebra* including real numbers without strict treatment. Arithmetical and geometrical series are considered, as well as the binomial theorem and the method of mathematical induction. The course does *not* include Euclid's algorithm, nor any theorems of the elementary number theory. The imaginary unit is mentioned but only in passing, and no general treatment of complex numbers is given. *Geometry* in the plane and solid geometry corresponding to about Euclid's elements. Plane trigonometry and analytical geometry in the plane based on right-angled systems of coordinates. Straight lines. Conic sections, regarded as curves of the second degree without a product term. The course does *not* include projective and general affine geometry, nor transformations of the coordinates

beyond translations and simple changes of the axes. *Theory of functions of one variable*: The concepts of functions, limits of functions, continuity and derivative with proof of existence for rational, circular and simple algebraic functions. Application especially to extremum problems, graphic representation of functions, their tangents and normals. Integration only as inversion to derivation with calculation of areas and volumes. The treatment depends to a high degree on intuition and no deeper theorems are proved. This is especially the case as to the fundamental notion of limit which is introduced disregarding any questions of stringency. The course does *not* include the definite integral, nor any treatment of the inverse circular functions. The derivative of the exponential and logarithmic functions has hitherto not been treated, but for the future there is at least a scheme for taking up this study. *Theory of probability and statistics*: This item is entirely new in Swedish schools, introduced only two years ago when the new plan of education was passed. Some of the fundamental notions such as mean, standard deviation, error, and correlation will be mentioned; however, the judgment of the result must be postponed until 1957 when the matriculation examination according to the new plan will be held for the first time.

As I have said before, the course of mathematics just outlined is a maximal one, studied only by those who choose the mathematical branch of the science side—their number might be about 15% of the total number of the top-form pupils. In the biological branch the course is much reduced, especially concerning analytical and solid geometry, but also concerning the theory of functions. For instance, the only functions studied with their derivatives are polynomials. Perhaps, there may be added in the future the exponential function and its inverse. The reduction of the geometrical items in this branch is quite natural, however, the reduction of the theory of functions appears to be more doubtful.

Comparing the course of mathematics in Swedish schools with what is studied in most other European countries, also in our neighbouring countries, Denmark and Norway, one gets the impression that we do not get very far, and a foreigner might

perhaps imagine that mathematics in Sweden is a subject which should certainly not give trouble to young men and women, nor to their mothers. And yet, to public opinion mathematics is considered a very hard subject, and it is true that it gives rise to a great percentage of low marks. One might ask why. Of course, a general answer is that even a very modest course of mathematics needs a certain power of abstraction and imagination which only relatively few possess. But this answer does not get under the skin of the problem, and to have a more exhaustive one it is necessary to consider our special form of examination.

The result of the teaching during a certain period, say a month, is examined by means of a written test consisting of a number of problems, generally eight, which the pupil has to solve under supervision. Upon the whole, the number of problems solved decides the point given, three problems being necessary for a pass mark. If the written tests during a term have been passed, the pupil is generally admitted irrespective of what else he may show up. If, on the other hand, the written tests are bad, he has only a poor chance of compensating this shortcoming by doing well during the oral lessons. Now, obviously there are hard problems even in a meagre theory; in reality, the more meagre the theory is, the more difficult the problems turn out to be because of the pupils' lack of outlook and because of a natural lack of problems which are not too trivial. To supply this want problems combining different parts of the course are constructed, and though such combinations may be valuable in principle, the constructions of such problems very often assume shapes which are quite grotesque. And so, a less talented or nervously inclined pupil may fail even if the theoretical knowledge demanded is very modest.

The final examination, the matriculation examination is taken in a similar way. Every year in March a paper is set, common to the whole country, the problems being ultimately selected by the Board of Education. For the pupils admitted, an oral examination takes place in May. Besides by the teacher this exam is also judged by censors, nominated ad hoc and provided with authority to approve or to reject. However, also

in this exam the results of the written tests during the last two terms play an important part. If these have been passed the pupil has nothing much to fear; a failure may be accounted for in one way or another, e. g. as being due to a nervous breakdown such as " Well, yes, things went amiss today but he is generally much better, probably he has not slept well, and, in any case his written tests are fairly good, so what ? " If, on the other hand, the written tests are bad, a good oral exam may help the examinee to get through, but in many cases he will not obtain a pass mark in mathematics.

It stands to reason, under these circumstances with this very strong emphasis on the written tests, that all the ambition and all the efforts of the teacher should aim at the one great task: to teach his pupils how to solve problems. This method of education, properly carried through, may lead a step further, at least on a more advanced level: how to pose problems. If the mathematical master has reached so far, I think he has been successful, for then he has got the very nucleus of mathematics and he is following, in a humble way, the road trodden by all great mathematicians. I am therefore inclined to believe that there is a very sound idea underlying our system of education and examination, which places the problems in the foreground and the theory as a background.

But a good idea alone does not secure progress. Solving a problem is applying step by step the laws of arithmetic, the geometrical intuition or the knowledge of some simple functions. A talented youth could do all this by simple reasoning without much practice, but very few will succed in this way. Most pupils will need practice, again and yet again, and the more of it the more complicated the problems are. Even the best pupils need much practice in order to be able to solve all the problems in a test, the time being limited. Then, what sort of problems should the teacher select for the training of his pupils ? Of course, he is free to choose what he likes within the limits drawn up by his own teaching, but if he aims at good results in the official examination-paper, he will be kind enough to choose the types of problems that in all probability will appear there. And if he is pressed for time, which he always is, and if, con-

sequently, he has to dash along over some part of the course, which part could he most conveniently neglect ? Of course the part which does not appear in the form of problems in the examination-paper ! He might even, if he is not too ambitious, try a somewhat dangerous but fully practicable way: to leave mathematics in the strict sense behind and teach *methods* to solve different types of problems. The theory, which should give the pupils a certain idea of what mathematics is, is then very often relegated to the time of a month or so between the written and oral examination. I remember the story of an old teacher, disillusioned by many years' not too gifted pupils, who at an unexpected " why " simply answered " otherwise it would be wrong ". In this way the examination-paper, and as a whole this alone, is decisive of the range of mathematical education, independent of what is stipulated in the general plan. There is the danger that an item that is not to be represented among the problems given might in the long run be neglected. On the other hand, during the last decade, for instance, great stress has been laid on polynomial or rational curves depending on a parameter because of their repeated appearance in the examination-paper; in my opinion much more stress than they deserve from a mathematical point of view. An example: " Consider the curve $10y = x^4 - 8x^3 + 2ax^2$ for different values of the constant a, especially with regard to the extremum values, and sketch the different types of graphs which may occur." This problem is not very difficult, nor very interesting, but it needs the construction of six curves, only slightly differing from each other.

As I already mentioned a new plan of education was decided upon two years ago. This plan might be described as a cautious revision of the old one; some new important items have been introduced and a little of the old stuff has been removed. One might have preferred a more radical reform, but the law of inertia also seems to rule mathematics, let alone the mathematicians. For one must always bear in mind that the great majority of teachers, with years of practice and experience, are not so easily reformed by a simple stroke of a pen. What he used to do he does, and news is not swallowed voraciously.

Because of this continuity is important. In any case, a step forward has been made and the foundation laid for further development.

But the system of examination remains the same and so we are back at the examination-paper. Without a rather thorough reform of this paper and perhaps of the method of calculating the marks all efforts to bring new life into mathematical education are doomed to fail. Considering the increasing number of pupils in a form and their generally decreasing elementary knowledge, there will never be time for any new items, nor for any deepening of the ones already existent, if the special training for solving special problems must be maintained. The problems in view are distinguished not so much by their difficulty—in principle they are rather harmless—but by the amount of work they require. This year (1955) the examination-paper seemed rather hard and I presumed at first that the result would be bad. I was wrong; it was quite normal. The training had been good. Yet, there wouldn't be much to say, generally speaking, about this training, if it had had as a result some deeper mathematical knowledge, but this is generally not the case, the domain being too limited and in a certain sense closed.

Now I suppose that these difficulties in mathematical education, which I have described, are, more or less, to be found everywhere, and therefore I have bothered you with this report in the hope of having a discussion and of hearing opinions based on experiences made in other countries. I have certain ideas as to the direction in which a tolerably good solution might be found, but I should like to hear your opinion.

1. We cannot afford to give up the written tests, nor the examination-paper. Mathematics is perhaps in a higher degree than other school subjects a subject suited for personal activity on the part of the pupils, and they must needs become accustomed to doing a bit of work without any assistance. Moreover, the different items in mathematics depend on one another so intimately, that without written tests the teacher would always be in a quagmire.

2. In order to reduce the time necessary for practice, which practice may, for the better pupils, prove rather spiritless, and

to get time to discuss more interesting things, about two thirds of a written test and of course of the examination-paper should consist of quite simple problems, meant only for the control of knowledge, so that even an average pupil might easily get pass mark without hard training.

3. There should be no limitation in the examination-paper as to the items represented among the problems. Experience has shown that limitations bring about negligence of the parts left out whereas complicated monsters of problems are constructed on the remaining parts.

4. The dependency of the mark on the written tests and on the examination-paper should be made less decisive and more consideration should be given to knowledge or lack of knowledge shown in other ways. Admittedly, this is a delicate matter which puts a good deal of responsibility on the teacher and, in the final exam, also on the censor, but with a certain amount of common sense difficulties should be overcome.

Résumé

Les cours de mathématiques dans les écoles secondaires de Suède ne sont pas très étendus, et, dans plusieurs cas, l'exposition de la matière manque de rigueur. D'un autre côté, la solution des problèmes est enseignée minutieusement. Cela tient en grande partie à ce que les examens écrits, qui sont composés d'un nombre de problèmes à résoudre, sont presque entièrement décisifs pour la note donnée. Il en est ainsi en particulier de l'examen du baccalauréat. Il serait préférable de faire les cours théoriques un peu plus substantiels en élargissant le domaine d'instruction aussi bien que par un certain approfondissement. Le commencement a été fait déjà par le nouveau plan d'études, passé il y a deux ans. Or, pour gagner le temps nécessaire, il sera inévitable de réformer assez radicalement les examens écrits et peut-être aussi la méthode pour calculer les notes. Un projet d'une telle réforme est déposé.