THE FUNCTION OF MATHEMATICS IN MODERN SOCIETY AND ITS CONSEQUENCE FOR THE TEACHING OF MATHEMATICS [1]

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1. SOCIETY'S GROWING DEMAND FOR MATHEMATICS.

The degree to which mathematics are applied to other sciences and to non-scientific social activities is rapidly increasing, in particular during the last decades: any convenient mathematical model for it would have to have positive derivatives, at least of the first and second order.

Firstly the number of fields to which mathematics are applied increases. To the classical fields: astronomy and geodesy,

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mechanics, physics, technical and actuarial sciences, later biology, economy and psychology have been added, at first in the form of biometry, econometry and psychometry, mostly using statistical methods. More recently such methods are more and more applied to industrial planning, to medicine, biochemistry, physiology and pharmacology, to sociology, cryptology, etc.

Also philosophy and even linguistics (mechanical translation), get slowly interested in applying mathematical and symbolic logical and semantical methods. It is a curious fact that, although often a first initiative was taken by mathematicians, it is on the whole not due to mathematical propaganda and advertising, but rather to genuine autonomous demand from the side of the workers in these different fields, which feel more and more helpless if they cannot handle the mathematical methods themselves. Only a few domains have abandoned the use of mathematics, in particular music and the pictorial arts (perspective). Whether to their advantage or not, more competent judges may decide. Among the fields which have hardly begun to make use of mathematical and logical methods occurs, surprisingly, the teaching of mathematics.

Also the number and the variety of applications of mathematics have greatly increased. Extensive new branches have been created which are wholly or mainly based on mathematical methods. As such we mention, leaving aside the classical fields of physics and astronomy [1]: Design of experiments, in particular the analysis of variance, at first mainly used in agriculture, later also in many other fields; Renewal theory in mathematical population theory; Theory of risk and net retain in insurance; Symbolic logic and semantics; Biomathematics; Factor analysis in psychology, etc.; Quality control; Mathematical theory of communication; Information theory and cybernetics; Econometric decision theory, based on the theory of strategic games, in particular linear programming; Periodogram-analysis and time series theory; Theory of statistical decision functions, etc.

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[1] Neither this nor any other of the further lists has any pretention of completeness.

Although we might not claim that the new theories in all cases yield a practical output equivalent to their mathematical difficulty, the judgment of the workers in these fields considers them on the whole as beneficial to their particular domain.

All this requires a re-orientation of the teaching of mathematics, in particular in secondary schools, towards which the present enquiry of CIEM may be considered as a decisive step.

2. Pure and applied mathematics.

Until a few decades ago applied mathematics was considered by the majority of mathematicians as second rank mathematics, notwithstanding the fact that almost all mathematicians till Laplace and Gauss, and since that time e.g. Riemann and Poincaré derived some of their most important results from the applications. This opinion expresses itself already in the word " pure " which is a (positive) " appraisal " according to Charles Morris' terminology, and is probably related to the then preponderant idealistic philosophy, mostly from German origin. It overrates greatly some special features of socalled " pure " mathematics, which, apart from a few branches like number theory and topology, almost all originated humbly from old applications (e.g. the theory of—in particular partial—differential equations and integral equations; Bessel-, Legendre-, and most other special functions). Applied mathematics seems to be like wine: it becomes pure just in course of time. With regard to mathematical rigour and generality modern applied mathematics need not be a second to the pure brand. In fact, mathematical rigour is often overdone in modern applications. A scientific theory then becomes a counterpart to the king's palace in the story of Aladdin's lamp: if a problem belongs to a scientific theory containing many points of considerable doubt and rough approximations, then to give a perfectly rigorous proof of existence of its solution in the mathematical part, is like building up one window of the palace wholly out of diamonds and rubies, whilst leaving all other ones made from plain glass.